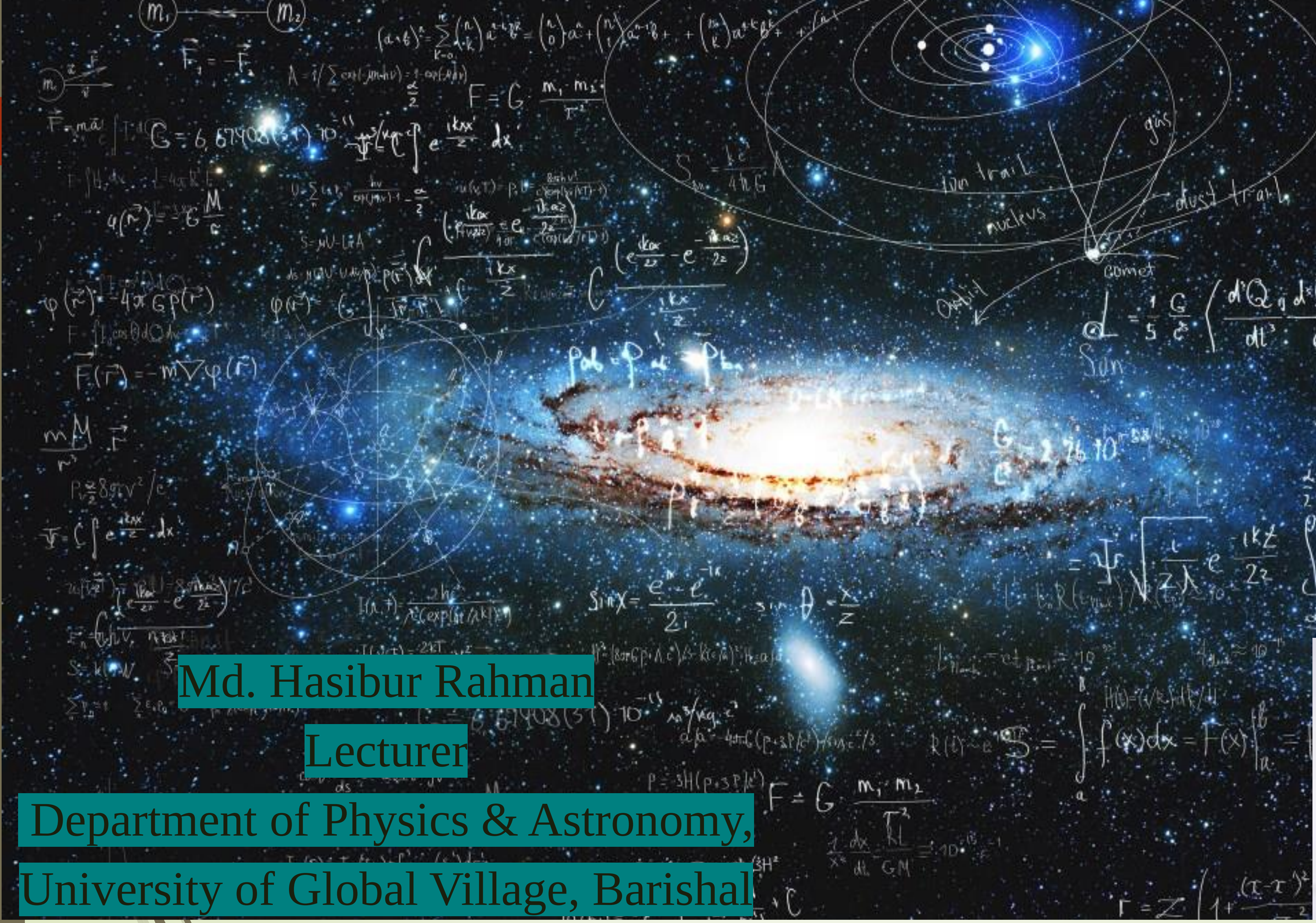
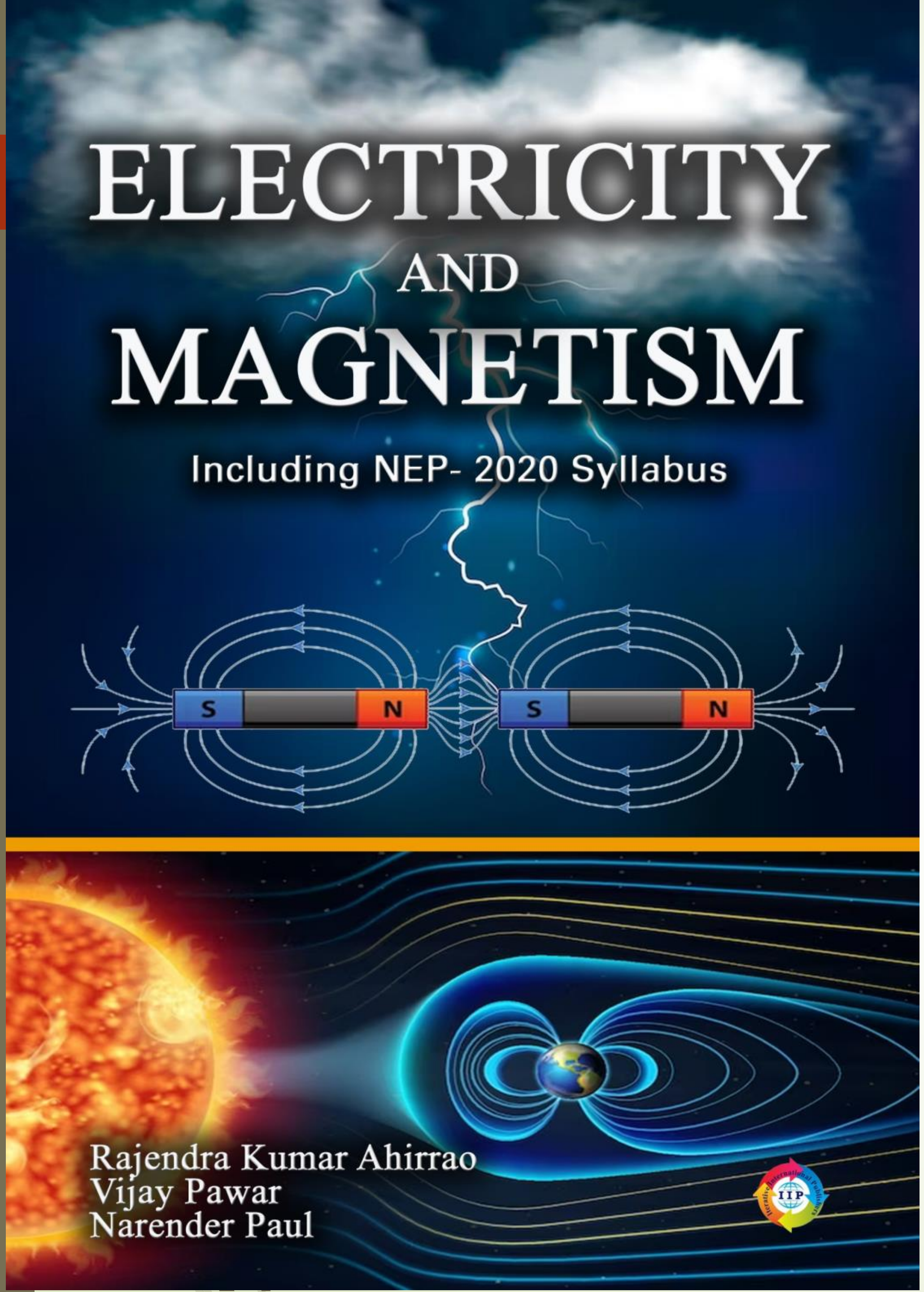


Md. Hasibur Rahman
Lecturer
Department of Physics & Astronomy,
University of Global Village, Barishal

UNIVERSITY OF GLOBAL VILLAGE



Course Title: Physics-I, Electricity, Magnetism & Properties of Matter

Rationale: This course is basic physics in the field of Electricity, Magnetism & Properties of Matter. The course will emphasize the fundamental concepts, and theories and solve quantitative problems that can be applicable to a wide spectrum of engineering disciplines.

Course Credit: 3

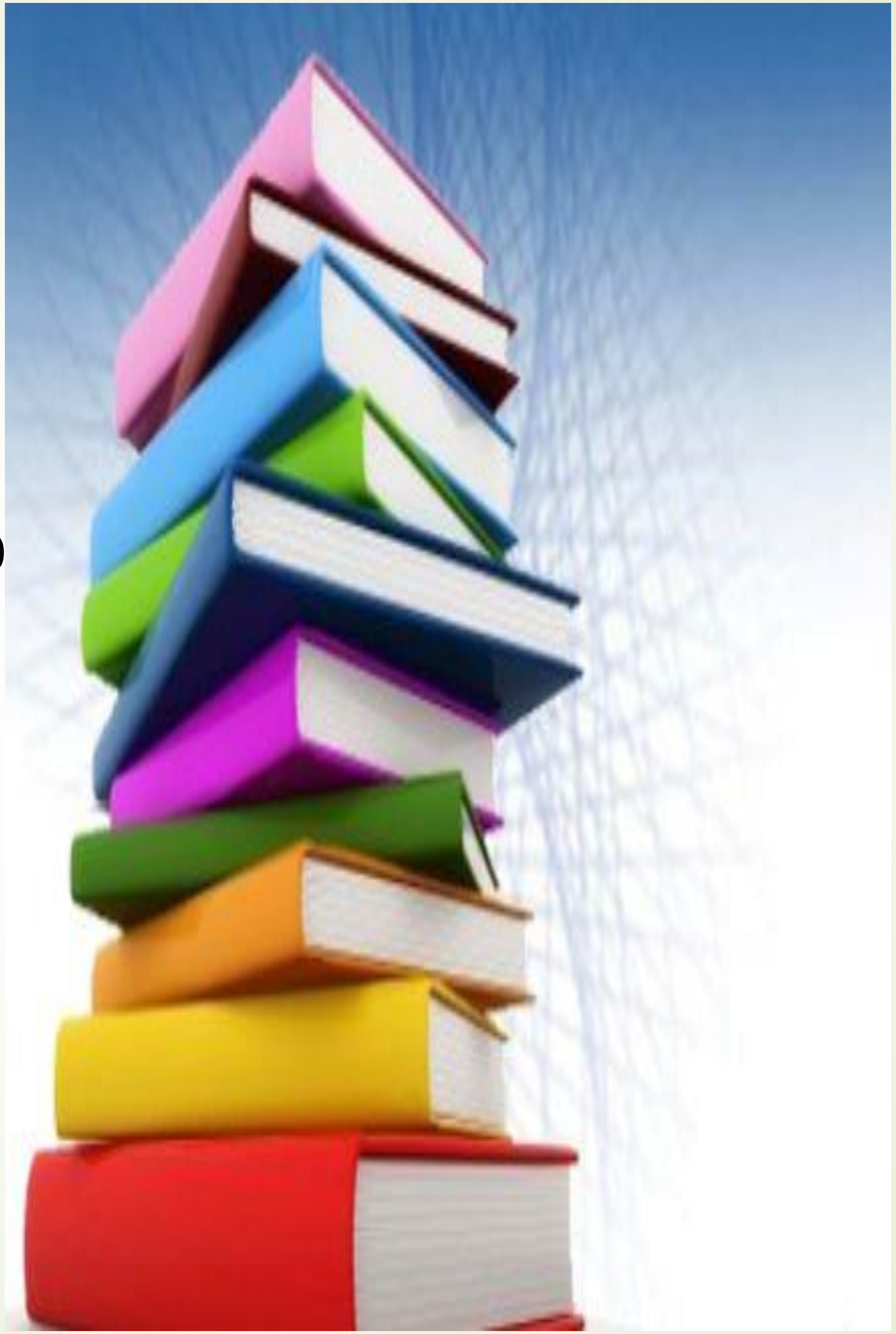
Course Code: PHY 0533-1101

Class: 17 Weeks (2 Lecture per week, Total = 34)

CIE Marks: 90

SEE Marks: 60

Total Marks: 150



Total: 26 weeks per semester

Exam/ Result: 06 weeks

Holiday/ Leave: 3 weeks

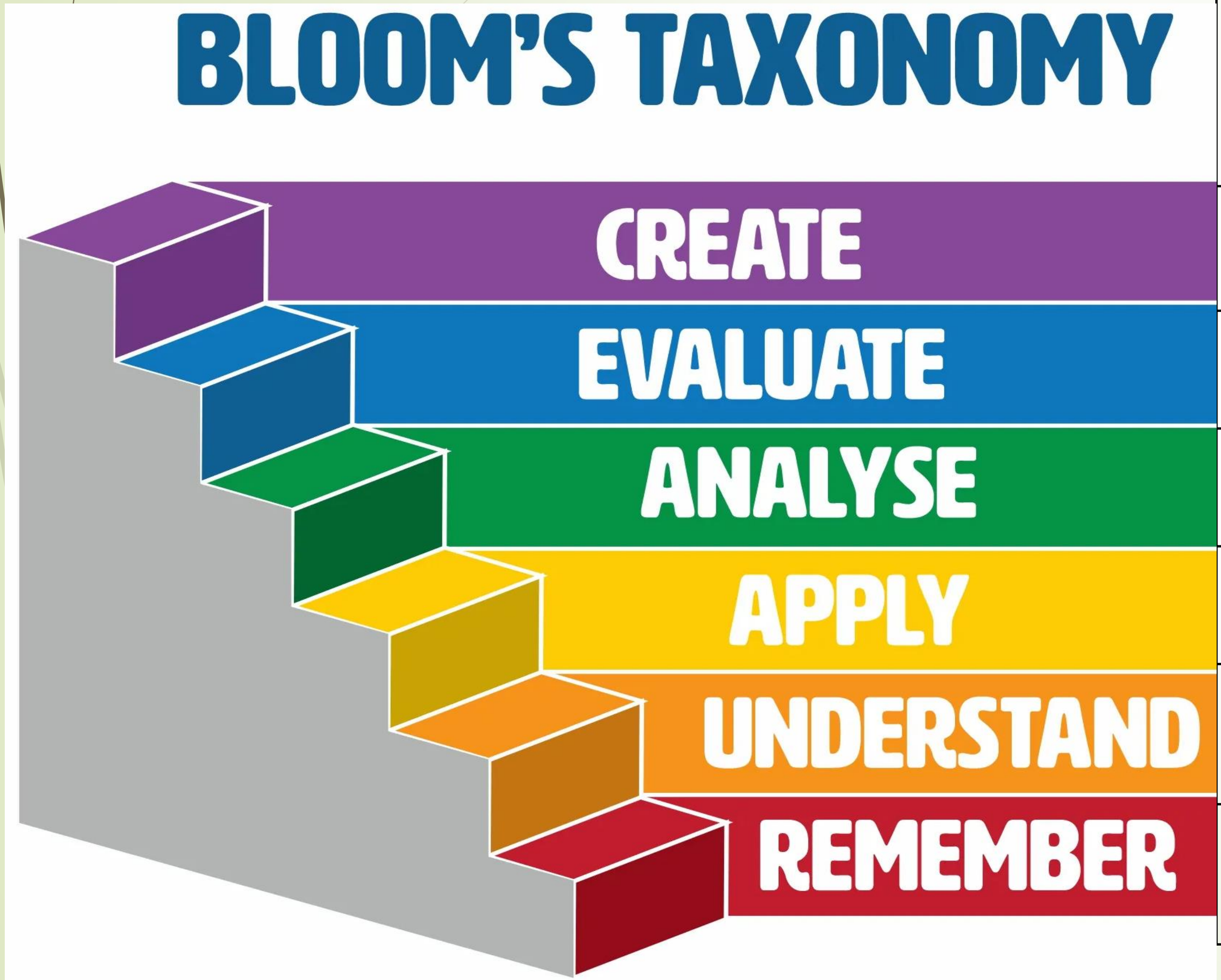
Assessment Pattern

Continuous Internal Evaluation (CIE 90 marks)

Blooms Category	Test (Out of 45)	Assignments (15)	Quiz (15)	Co-curricular Activities (15)
Remember	05		5	Attendance 15
Understand	05			
Apply	10			
Analysis	8	7	10	
Evaluate	7	8		
Create	10			

Assessment Pattern

Semester End Examination (SEE 60)



Test (Out of 60)
10
10
10
10
10
10

Grading System:

Numerical Grade	Letter Grade	Grade Point
80% and above	A+	4.00
75% to less than 80%	A	3.75
70% to less than 75%	A-	3.50
65% to less than 70%	B+	3.25
60% to less than 65%	B	3.00
55% to less than 60%	B-	2.75
50% to less than 55%	C+	2.50
45% to less than 50%	C	2.25
40% to less than 45%	D	2.00
Less than 40%	F	0.00
	F* Failure I** Incomplete W*** Withdrawal R**** Repeat Y***** Audit	

Reference Books for Electricity & Magnetism; Properties of Matter

1. Principles of Physics by David Halliday, Jearl Walker, and Robert Resnick
2. Concepts of Modern Physics by Arthur Beiser
3. Introduction to Solid State Physics by Charles Kittel
4. The Feynman Lectures on Physics, Vol. I: by Richard P. Feynman, Robert B. Leighton, and Matthew Sands
5. Matter and Interactions by Ruth W. Chabay and Bruce A. Sherwood
6. Physical Chemistry by Peter Atkins and Julio de Paula

Course learning outcomes (CLO): After successful completion of all the courses students will be able to

CLO1:

Knowing about different basic parameters in the field of Electricity, Magnetism & Properties of Matter.

CLO2:

Explaining and analyzing different theories and formulas for Vector, Wave, Simple harmonic oscillator, Elasticity, Gravity, Fluid Mechanics, Electricity and Magnetism, etc.

CLO3:

Solving quantitative problems in the fields of Vector, Wave, Simple harmonic oscillator, Elasticity, Gravity, Fluid Mechanics, Electricity and Magnetism, etc.

CLO4:

Discover different types of devices and analyze different material properties based on the theories of physics.

SL.	Content of Courses	Hrs	CLO’s
1	Vector addition, Vector subtraction, Unit vector, Applications of Vector, wave, Types of Wave, Doppler Effect, Simple harmonic Oscillator Energy of the oscillator, Topic Related Problems.	8	CLO1, CLO3
2	Elasticity, Stress, Strain, Hook's Law, Elastic Modulus, Pascal's law, Buoyant force, Archimedes Principle, Continuity Equation, Bernoulli's Equation, Topic Related Problems.	10	CLO2, CLO3
3	Conductor, Insulator, Semiconductor, Coulomb’s Law, Electric Field, Electric Field lines, Electric Flux, Electric Potential Energy, Capacitor and Capacitance, Dielectrics, Gauss's Law, Topic Related Problems	8	CLO3, CLO4
4	Magnetism, Magnetic force on a conductor, the Hall effect, The Biot-Savart Law, Ampere's Law, Topic Related Problems, review and Discussion	8	CLO3, CLO4

No.	Course Outcome
CLO1	Knowing about different basic parameters in the field of Electricity, Magnetism & Properties of Matter.
CLO2	Explaining and analyzing different theories and formulas for Vector, Wave, Simple harmonic oscillator, Elasticity, Gravity, Fluid Mechanics, Electricity, and Magnetism etc.
CLO3	Apply problem-solving techniques to quantitative problems in the areas of electricity, magnetism, and the mechanical behavior of materials.
CLO4	Explore and evaluate electrical circuits, magnetic systems, and material properties using theoretical concepts and experimental methods.

Week	Topics	Teaching and Learning Strategy	Assignment Strategy	Corresponding CLOs
1	Vector addition, Vector subtraction, Unit vector and Applications of Vector Vectors Problems.	Lecture, Oral Presentation, Video, presentation	Quiz, Assignment, and Written Exam, Presentation	CLO1, CLO3
2	Simple harmonic Oscillator The energy of the oscillator, , Doppler effect, SHO-related Topic Related Problems.	Lecture, Oral Presentation,	Quiz, Assignment, and Written Exam	CLO1, CLO3
3	Elasticity, Stress, Strain Hooke's Law, Elastic Modulus, Poisson’s ratio.	Lecture, Oral Presentation	Quiz, Assignment, Written Exam, and Discussion	CLO1, CLO2

Week	Topics	Teaching and Learning Strategy	Assignment Strategy	Correspondin g CLOs
4	Newton's Law of Universal Gravitation, Kepler’s Law, and the motion of Planets, Topic Related Problems.	Lecture, Oral Presentation, Video presentation.	Quiz, Assignment, and Written Exam,	CLO1, CLO3
5	Energy consideration of the Planetary and satellite motion, Topic Related Problems	Lecture, Oral Presentation,	Quiz, Assignment, and Written Exam	CLO2, CLO3
6	Pascal's Law, Archimedes Principle, Torricelli experiment, Fluid Dynamics, Topic Related Problems,	Lecture, Oral Presentation	Quiz, Assignment, Written Exam, and Discussion	CLO3, CLO4

Week	Topics	Teaching and Learning Strategy	Assignment Strategy	Corresponding CLOs
7	Ideal fluid and Bernoulli’s Principle Continuity Equation, Topic Related problems,	Lecture, Oral Presentation,	Quiz, and Written Exam,	CLO1, CLO3
8	Band Theory, Conductor, Insulator, Semiconductor, Coulomb’s Law, Electric Field, Electric Field lines,	Lecture, Oral Presentation,	Quiz, Assignment, and Written Exam	CLO2, CLO3
9	Electric Flux, Electric Potential Energy, Capacitor and Capacitance, Topic Related problems	Lecture, Oral Presentation	Quiz, and Written Exam	CLO1, CLO2
10	Dielectrics and piezoelectricity, Kirchhoff's Current Law (KCL), Kirchhoff's Voltage Law (KVL), Topic Related problems	Lecture, Oral Presentation and Video presentation	Quiz, and Written Exam	CLO1, CLO2

Week	Topics	Teaching and Learning Strategy	Assignment Strategy	Corresponding CLOs
11	Gauss's Law, Magnetism, Magnetic force on a conductor, Magnetism related problems	Lecture, Oral Presentation, Video presentation.	Quiz, Assignment, and Written Exam,	CLO3, CLO4
12	Magnetic Domains and Hysteresis: Origin of magnetism,	Lecture, Oral Presentation,	Quiz, and Written Exam	CLO2, CLO3
13	Different types of magnetism Hall effect, Topic Related Problems	Lecture, Oral Presentation	Quiz, and Written Exam	
14	The Biot-Savart Law, Ampere's Law Topic Related Problems.	Lecture, Oral Presentation and video presentation	Quiz, and Written Exam	CLO1, CLO2

Week	Topics	Teaching and Learning Strategy	Assignment Strategy	Corresponding CLOs
15	Maxwell's fundamental laws, Topic Related Problems.	Lecture, Oral Presentation, Video presentation.	Quiz, Assignment, and Written Exam,	CLO1, CLO3
16	Review and discussion class	Lecture, Oral Presentation,	Quiz	CLO3
17	Review and discussion class	Lecture, Oral Presentation,	Quiz	CLO3

1ST WEEK

TOPIC:

PROPERTIES OF MATTER:

PHYSICAL QUANTITY,

VECTOR ANALYSIS,

TOPIC RELATED MATH

PAGE: 16- 37

❖ What is Matter

Matter is defined as anything that has mass and occupies space. It is composed of particles, primarily atoms and molecules, which have mass and volume.

❖ What is Physical Quantity?

The physical quantities in physics are what we can measure or sense in an object or a phenomenon.

Physical quantities can be categorized into two main types: scalar and vector quantities. Additionally, there are derived quantities and fundamental quantities.

1. Scalar Quantities:

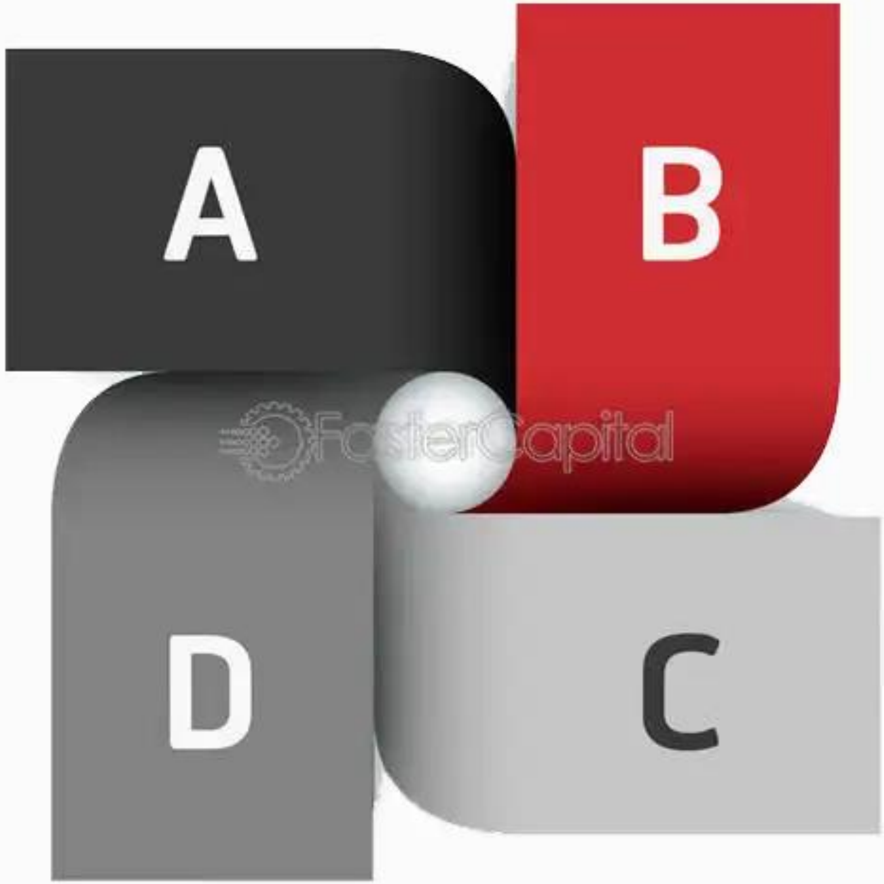
Scalar quantities are described by their magnitude only, without any associated direction. They are fully characterized by a numerical value and a unit. Examples of scalar quantities

Mass (e.g., 5 kg), Temperature (e.g., 30°C), Time (e.g., 10 seconds), Energy (e.g., 200 joules)

Real World Applications of Vector Calculations

Motion Planning in Robotics

Aerospace Engineering



Navigation

Structural Analysis

2. Vector Quantities:

Vector quantities have both magnitude and direction, and they follow the rules of vector algebra. Vector quantities are represented by arrows, where the length of the arrow represents the magnitude, and the direction of the arrow represents the direction of the vector. Examples of vector quantities include:

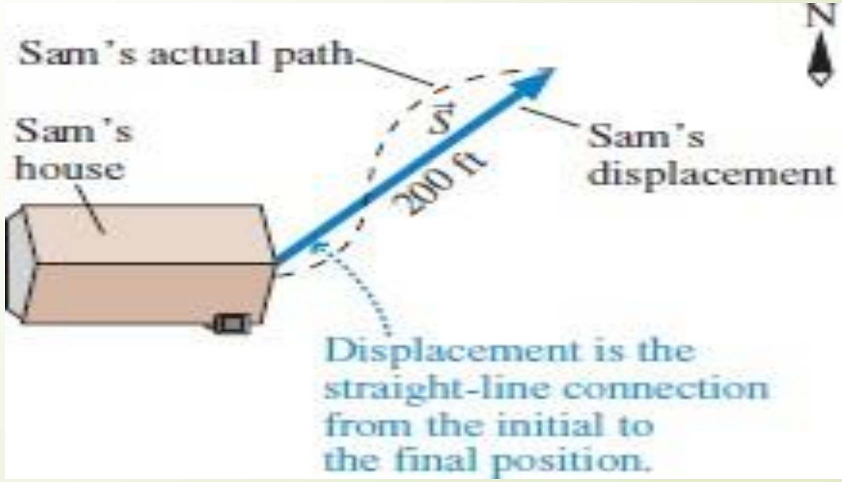
•Displacement (e.g., 10 meters east)

•Velocity (e.g., 20 m/s north)

•Force (e.g., 50 Newtons upward)

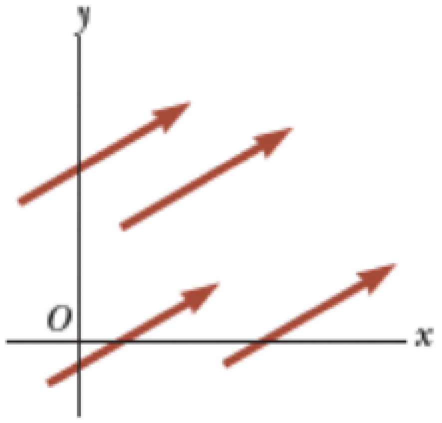
•Displacement

Suppose Sam starts from his front door, walks across the street, and ends up 200 ft to the northeast of where he started. Sam's displacement is a vector quantity. But, Sam's actual path is a Scalar quantity.



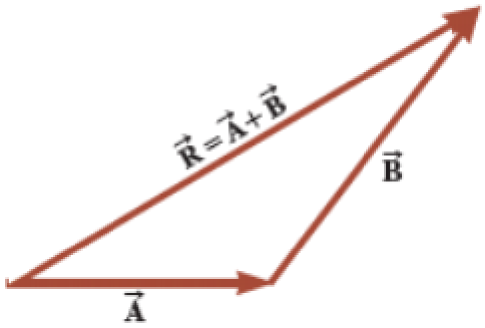
Some Properties of Vectors

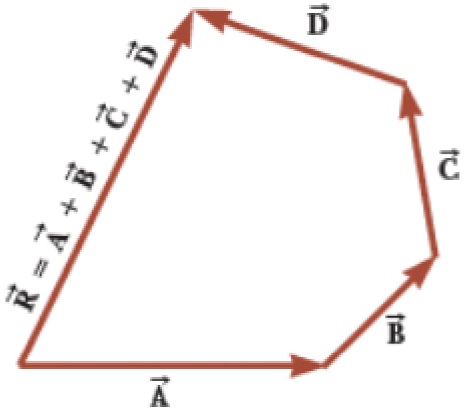
- **Equality of Two Vectors**



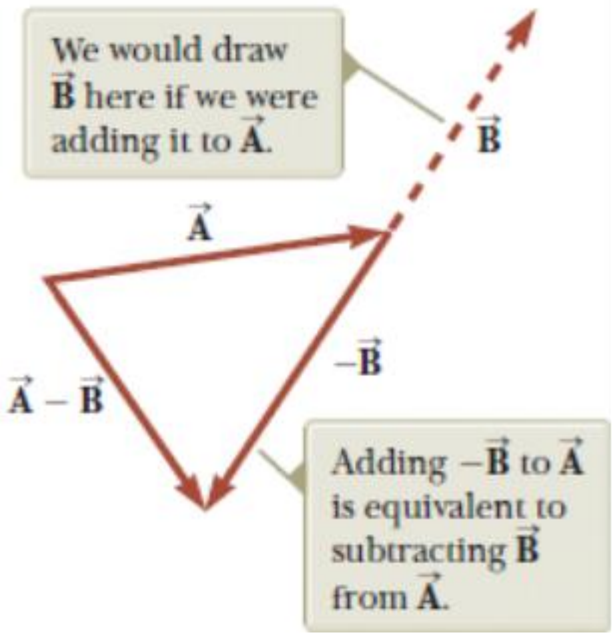
These four vectors are equal because they have equal lengths and point in the same direction.

- **Adding Vectors**





- **Subtracting Vectors**

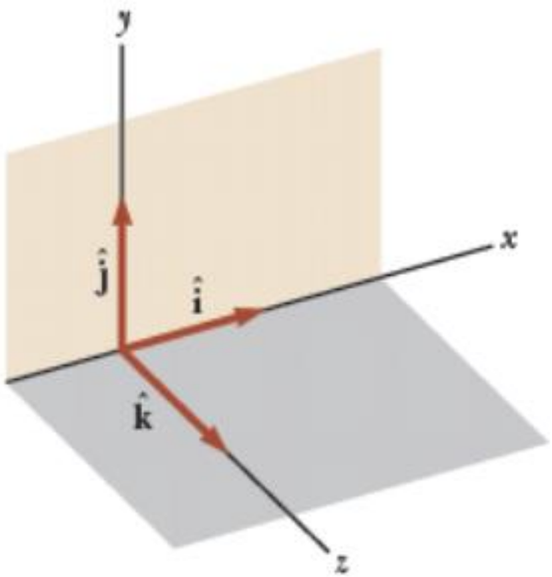


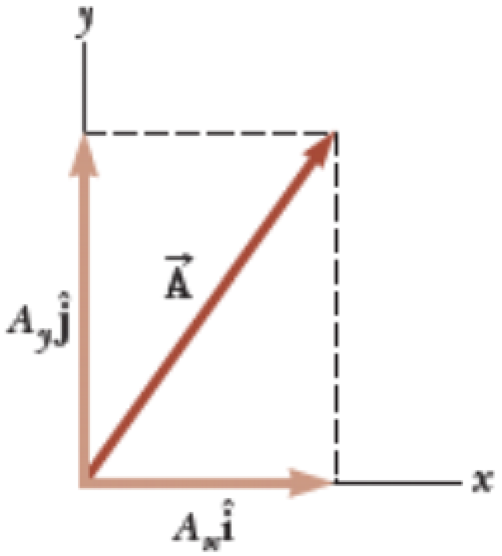
Unit Vectors

A **unit vector** is a dimensionless vector having a magnitude of exactly 1. Unit vectors are used to specify a given direction and have no other physical significance.

We shall use the symbols $i\Sigma$, $j\Sigma$ and k to represent unit vectors pointing in the

respectively.





$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

The sum of $\vec{A}$ and $\vec{B}$ is

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

Find the sum of two displacement vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ lying in the *xy* plane and given by

$$\vec{\mathbf{A}} = (2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\text{ m} \quad \text{and} \quad \vec{\mathbf{B}} = (2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}})\text{ m}$$

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (2.0 + 2.0)\hat{\mathbf{i}}\text{ m} + (2.0 - 4.0)\hat{\mathbf{j}}\text{ m} \qquad R_x = 4.0\text{ m} \qquad R_y = -2.0\text{ m}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0\text{ m})^2 + (-2.0\text{ m})^2} = \sqrt{20}\text{ m} = 4.5\text{ m}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{-2.0\text{ m}}{4.0\text{ m}} = -0.50 \qquad \theta = 333°$$

A particle undergoes three consecutive displacements: $\Delta\vec{\mathbf{r}}_1 = (15\hat{\mathbf{i}} + 30\hat{\mathbf{j}} + 12\hat{\mathbf{k}})$ cm, $\Delta\vec{\mathbf{r}}_2 = (23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}})$ cm, and $\Delta\vec{\mathbf{r}}_3 = (-13\hat{\mathbf{i}} + 15\hat{\mathbf{j}})$ cm. Find unit-vector notation for the resultant displacement and its magnitude.

$$\Delta\vec{\mathbf{r}} = \Delta\vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_3 = (15 + 23 - 13)\hat{\mathbf{i}}\text{ cm} + (30 - 14 + 15)\hat{\mathbf{j}}\text{ cm} + (12 - 5.0 + 0)\hat{\mathbf{k}}\text{ cm}$$

$$= (25\hat{\mathbf{i}} + 31\hat{\mathbf{j}} + 7.0\hat{\mathbf{k}})\text{ cm}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(25\text{ cm})^2 + (31\text{ cm})^2 + (7.0\text{ cm})^2} = 40\text{ cm}$$

Real-Life Applications of Vector-Related Phenomena

❖ What is the Parallelogram Law of Vector Addition?

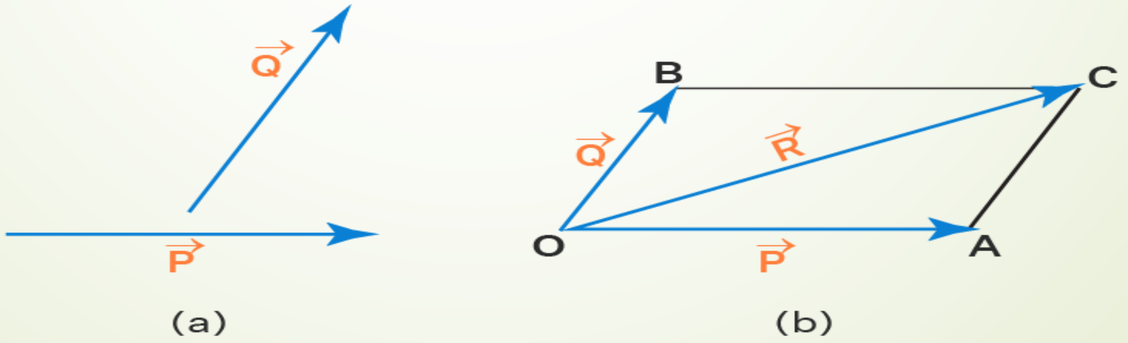
The parallelogram law of vector addition is the process of adding vectors geometrically. This law says, "Two vectors can be arranged as adjacent sides of a parallelogram such that their tails attach with each other and the sum of the two vectors is equal to the diagonal of the parallelogram whose tail is the same as the two vectors.

Consider the vectors P and Q in the figure below. To find their sum:

Step 1: Draw the vectors P and Q such that their tails touch each other.

Step 2: Complete the parallelogram by drawing the other two sides.

Step 3: The diagonal of the parallelogram that has the same tail as the vectors P and Q represents the sum of the two vectors. i.e., P + Q = R. Here, the vector **R** is called the resultant vector (of **P** and **Q**).



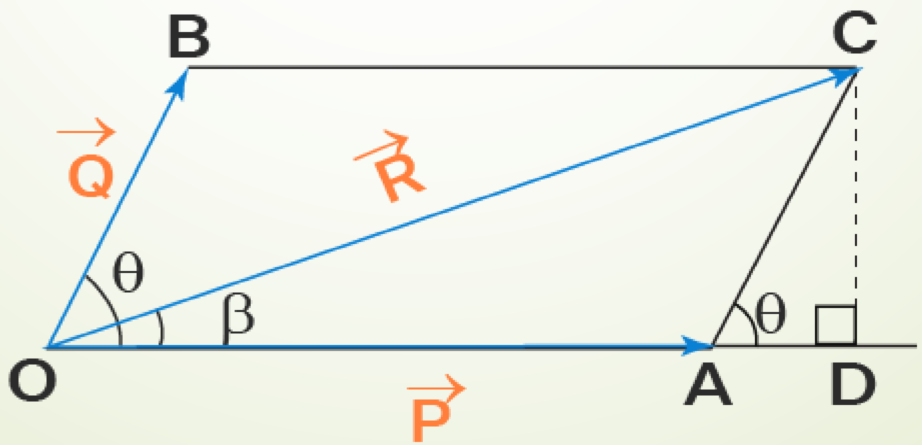
$\vec{R} = (\vec{P} + \vec{Q})$

❑ Parallelogram Law of Vector Addition Proof:

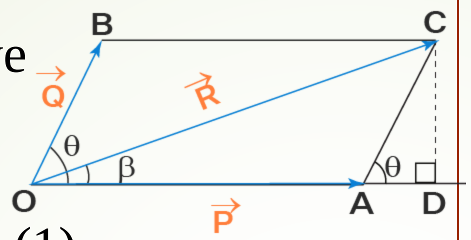
Now, to prove the formula of the parallelogram law, we consider two vectors **P** and **Q** represented by the two adjacent sides OB and OA of the parallelogram OBCA, respectively. The angle between the two vectors is θ. The sum of these two vectors is represented by the diagonal drawn from the same vertex O of the parallelogram, the resultant sum vector **R** which makes an angle β with the vector **P**.

Extend the vector **P** till D such that CD is perpendicular to OD. Since OB is parallel to AC, therefore the angle AOB is equal to the angle CAD as they are corresponding angles, i.e., angle CAD = θ. Now, first, we will derive the formula for the magnitude of the resultant vector **R** (side OC). Note that

- |**P**| = P
- |**Q**| = Q
- |**R**| = R



In right-angled triangle OCD, by Pythagoras' theorem, we have



$OC^2 = OD^2 + DC^2$

$\Rightarrow OC^2 = (OA + AD)^2 + DC^2$ --- (1)

In the right triangle CAD, we have

$\cos\theta = AD/AC$ and $\sin\theta = DC/AC$

$\Rightarrow AD = AC\cos\theta$ and $DC = AC\sin\theta$

$\Rightarrow AD = Q\cos\theta$ and $DC = Q\sin\theta$ --- (2)

Substituting values from (2) in (1), we have

$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$

$\Rightarrow R^2 = P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$

$\Rightarrow R^2 = P^2 + 2PQ\cos\theta + Q^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow R^2 = P^2 + 2PQ\cos\theta + Q^2$ [$\cos^2\theta + \sin^2\theta = 1$]

$\Rightarrow R = \sqrt{(P^2 + 2PQ\cos\theta + Q^2)}$

The magnitude of the resultant vector **R**.

Next, we will determine the direction of the resultant vector. We have in the right triangle ODC,

$\tan\beta = DC/OD$

$\Rightarrow \tan\beta = Q\sin\theta/(OA + AD)$ [From (2)]

$\Rightarrow \tan\beta = Q\sin\theta/(P + Q\cos\theta)$ [From (2)]

$\Rightarrow \beta = \tan^{-1}\left[\frac{(Q\sin\theta)}{(P + Q\cos\theta)}\right]$

→ Direction of the resultant vector **R**

❖ Special Cases of Parallelogram Law of Vector Addition

Now, we know the formula to determine the magnitude and direction of the sum of the two vectors. Let us consider a few special cases and substitute the values in the formula:

➢ When the Two Vectors are Parallel (Same Direction)

If vectors **P** and **Q** are parallel, then we have θ = 0°. Substituting this in the formula of the parallelogram law of vectors, we have

$|\mathbf{R}| = R = \sqrt{(P^2 + 2PQ \cos 0 + Q^2)}$

$= \sqrt{(P^2 + 2PQ + Q^2)}$ [Because cos 0 = 1]

$= \sqrt{(P + Q)^2}$

$= P + Q$

$\beta = \tan^{-1}[(Q \sin 0)/(P + Q \cos 0)]$

$= \tan^{-1}[(0)/(P + Q \cos 0)]$ [Because sin 0 = 0]

$= 0°$

- **When the Two Vectors are Acting in Opposite Direction**

If vectors **P** and **Q** are acting in opposite directions, then we have θ = 180°. Substituting this in the formula of parallelogram law of vector addition, we have

$|\mathbf{R}| = \sqrt{(P^2 + 2PQ \cos 180° + Q^2)}$

$= \sqrt{(P^2 - 2PQ + Q^2)}$ [Because cos 180°= -1]

$= \sqrt{(P - Q)^2}$

$= P - Q$

$\beta = \tan^{-1}[(Q \sin 180°)/(P + Q \cos 180°)]$

$= \tan^{-1}[(0)/(P + Q \cos 0)]$ [Because sin 180° = 0]

$= 0°$ or $180°$

- **When the Two Vectors are Perpendicular**

If vectors **P** and **Q** are perpendicular to each other, then we have θ = 90°. By the parallelogram law of vector addition, we have

$|\mathbf{R}| = \sqrt{(P^2 + 2PQ \cos 90° + Q^2)}$

$= \sqrt{(P^2 + 0 + Q^2)}$ [Because cos 90° = 0]

$= \sqrt{(P^2 + Q^2)}$

$\beta = \tan^{-1}[(Q \sin 90°)/(P + Q \cos 90°)]$

$= \tan^{-1}[Q/(P + 0)]$ [Because cos 90° = 0]

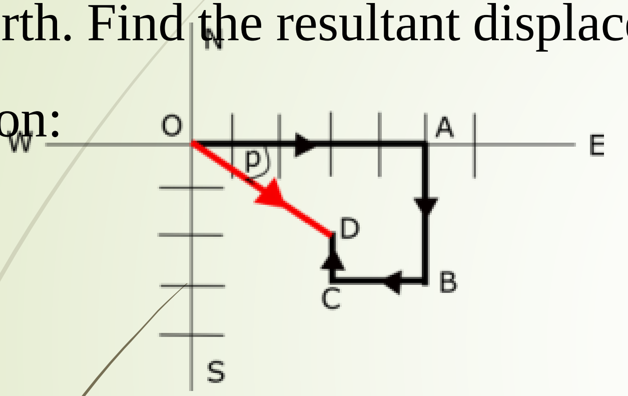
$= \tan^{-1}(Q/P)$

A car goes 5 km east 3 km south, 2 km west and 1 km north. Find the resultant displacement.

Example 1:

A car goes 5 km east 3 km south, 2 km west and 1 km north. Find the resultant displacement.

Solution:



First we will make the vector diagram

O to A 5 km east

A to B 3 km south

B to C 2 km west

c to D 1 km north

Net displacement is **OD**

Along the horizontal direction: 5 km east - 2 km west = 3 km east

Along the vertical direction: 3 km south - 1 km north = 2 km south

OD = $\sqrt{(3^2 + 2^2 + 2 \times 2 \times 3 \times \text{Cos } 90 \text{ deg})}$

= $\sqrt{(3^2 + 2^2)}$

= 3.6 km

tan p = 2/3

or p = $\tan^{-1} 2/3$ = 34 deg

Thus resultant displacement is 3.6 km, 34 deg south of east.

Example 2:

Two vectors P = (1, 2) and Q = (2, 4) have an angle of 0° between them. Find direction of the resultant vector and the magnitude of their sum vector.

Solution:

Using the parallelogram rule of vector addition formulas, we have

$|\mathbf{R}| = \sqrt{(P^2 + Q^2 + 2PQ \cos \theta)}$,

$\beta = \tan^{-1}[(Q \sin \theta)/(P + Q \cos \theta)]$

For this, first, we need the magnitudes of vectors **P** and **Q**.

$|\mathbf{P}| = \sqrt{(1^2 + 2^2)} = \sqrt{5}$, $|\mathbf{Q}| = \sqrt{(2^2 + 4^2)} = 2\sqrt{5}$

$|\mathbf{R}| = \sqrt{(\sqrt{5}^2 + (2\sqrt{5})^2 + 2PQ \cos \theta)}$

$= \sqrt{(5 + 20 + 2\times\sqrt{5}\times2\sqrt{5} \cos 0°)}$

$= \sqrt{(25 + 20)}$

$= \sqrt{(45)}$

$= 3\sqrt{5}$

Answer: The magnitude of the sum vector is $3\sqrt{5}$ units.

Two forces of magnitudes 4N and 7N act on a body and the angle between them is 45°. Determine the magnitude and direction of the resultant vector with the 4N force.

Example 3:

Two forces of magnitudes 4N and 7N act on a body and the angle between them is 45°. Determine the magnitude and direction of the resultant vector with the 4N force.

Solution:

Suppose vector **P** has magnitude 4N, vector **Q** has magnitude 7N, and $\theta = 45°$, then by the parallelogram law of vector addition:

$|\mathbf{R}| = \sqrt{(P^2 + Q^2 + 2PQ \cos \theta)}$

$= \sqrt{(4^2 + 7^2 + 2\times4\times7 \cos 45°)}$

$= \sqrt{(16 + 49 + 56/\sqrt{2})}$

$= \sqrt{(65 + 56/\sqrt{2})}$

≈ 12.008 N

$\beta = \tan^{-1}[(7 \sin 45°)/(4 + 7 \cos 45°)]$

$= \tan^{-1}[(7/\sqrt{2})/(4 + 7/\sqrt{2})]$

$\approx 28.95°$ **Answer:** The magnitude is approximately 12 N and the direction is 28.95°.

Assignment -01

1. Two forces of 3 N and 4 N are acting at a point such that the angle between them is 60 degrees. Find the resultant force
2. Find the resultant of the following two displacements: 2 m at 30 deg and 4 m at 120 deg. The angles are taken relative to the x-axis.
3. Two forces of 100N and 150N are acting simultaneously at a point. Find the resultant if the angle between them is 45^{o}

The Scalar Product of Two Vectors

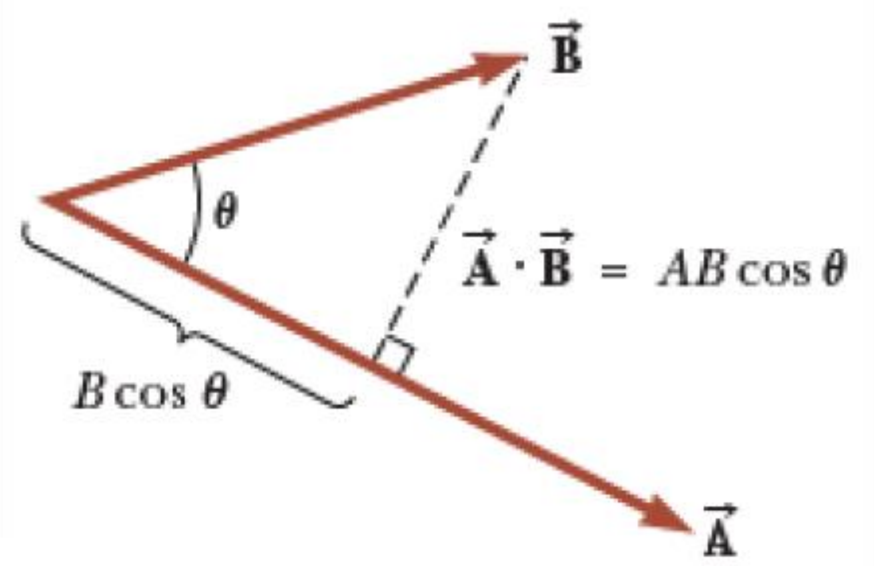
The scalar product of any two vectors A and $\vec{B}$ is defined as a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle u

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv AB\cos\theta$$

$$W = F\Delta r\cos\theta = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$$



$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{A}} = A_x^2 + A_y^2 + A_z^2 = A^2$$

The vectors $\vec{A}$ and $\vec{B}$ are given by $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = -\hat{i} + 2\hat{j}$.

(A) Determine the scalar product $\vec{A} \cdot \vec{B}$.

$$\vec{A} \cdot \vec{B} = (2\hat{i} + 3\hat{j}) \cdot (-\hat{i} + 2\hat{j})$$
$$= -2\hat{i} \cdot \hat{i} + 2\hat{i} \cdot 2\hat{j} - 3\hat{j} \cdot \hat{i} + 3\hat{j} \cdot 2\hat{j}$$
$$= -2(1) + 4(0) - 3(0) + 6(1) = -2 + 6 = 4$$

(B) Find the angle θ between $\vec{A}$ and $\vec{B}$.

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$
$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{65}} = 60.3^\circ$$

A particle moving in the xy plane undergoes a displacement given by $\Delta\vec{r} = (2.0\hat{i} + 3.0\hat{j})$ m as a constant force $\vec{F} = (5.0\hat{i} + 2.0\hat{j})$ N acts on the particle. Calculate the work done by $\vec{F}$ on the particle.

The Vector Product and Torque

$$\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$$

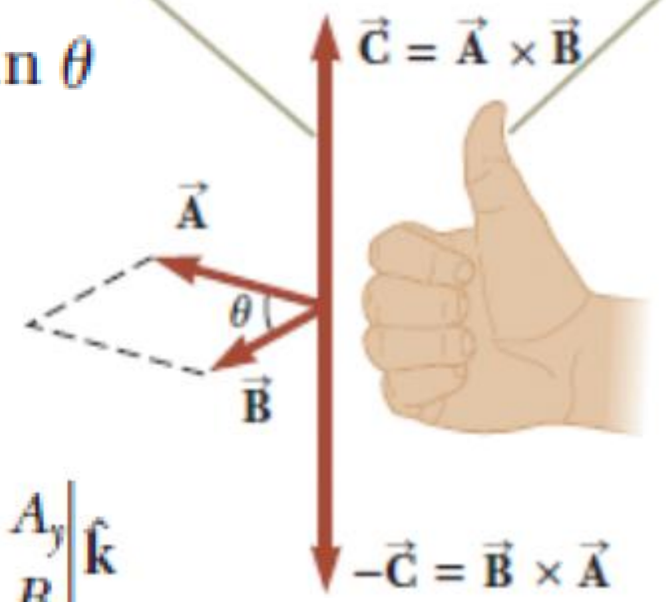
$$\hat{i}\times\hat{j}=-\hat{j}\times\hat{i}=\hat{k}$$

$$\hat{j}\times\hat{k}=-\hat{k}\times\hat{j}=\hat{i}$$

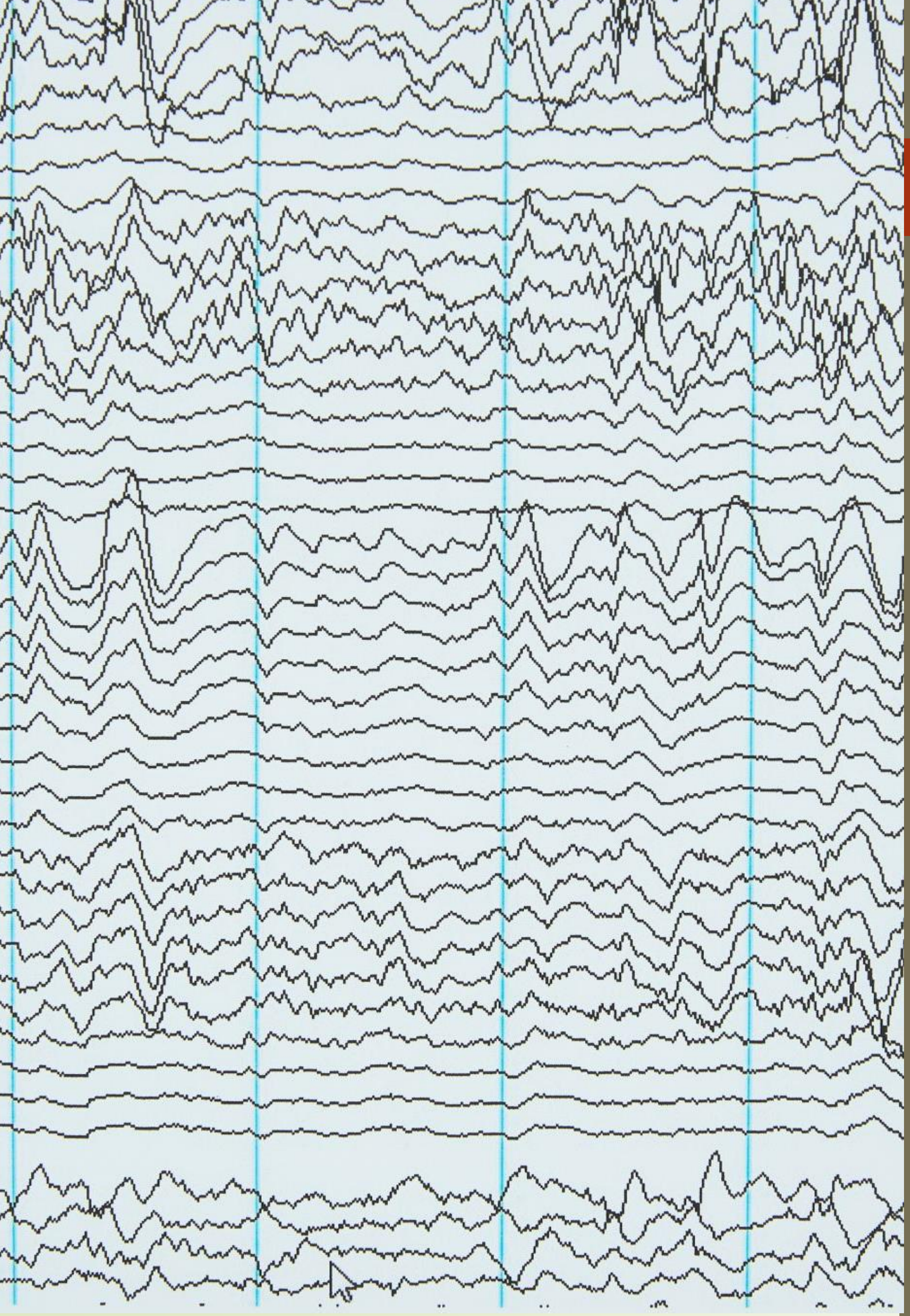
$$\hat{k}\times\hat{i}=-\hat{i}\times\hat{k}=\hat{j}$$

The direction of $\vec{C}$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$, and its direction is determined by the right-hand rule.

$$C = AB\sin\theta$$



$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}=\begin{vmatrix}A_y & A_z\\ B_y & B_z\end{vmatrix}\hat{i}+\begin{vmatrix}A_z & A_x\\ B_z & B_x\end{vmatrix}\hat{j}+\begin{vmatrix}A_x & A_y\\ B_x & B_y\end{vmatrix}\hat{k}$$



- **2nd Week**
- **Topic:**

Properties of Matter:

Wave & Oscillation,

Doppler Effect,

Topic Related Math

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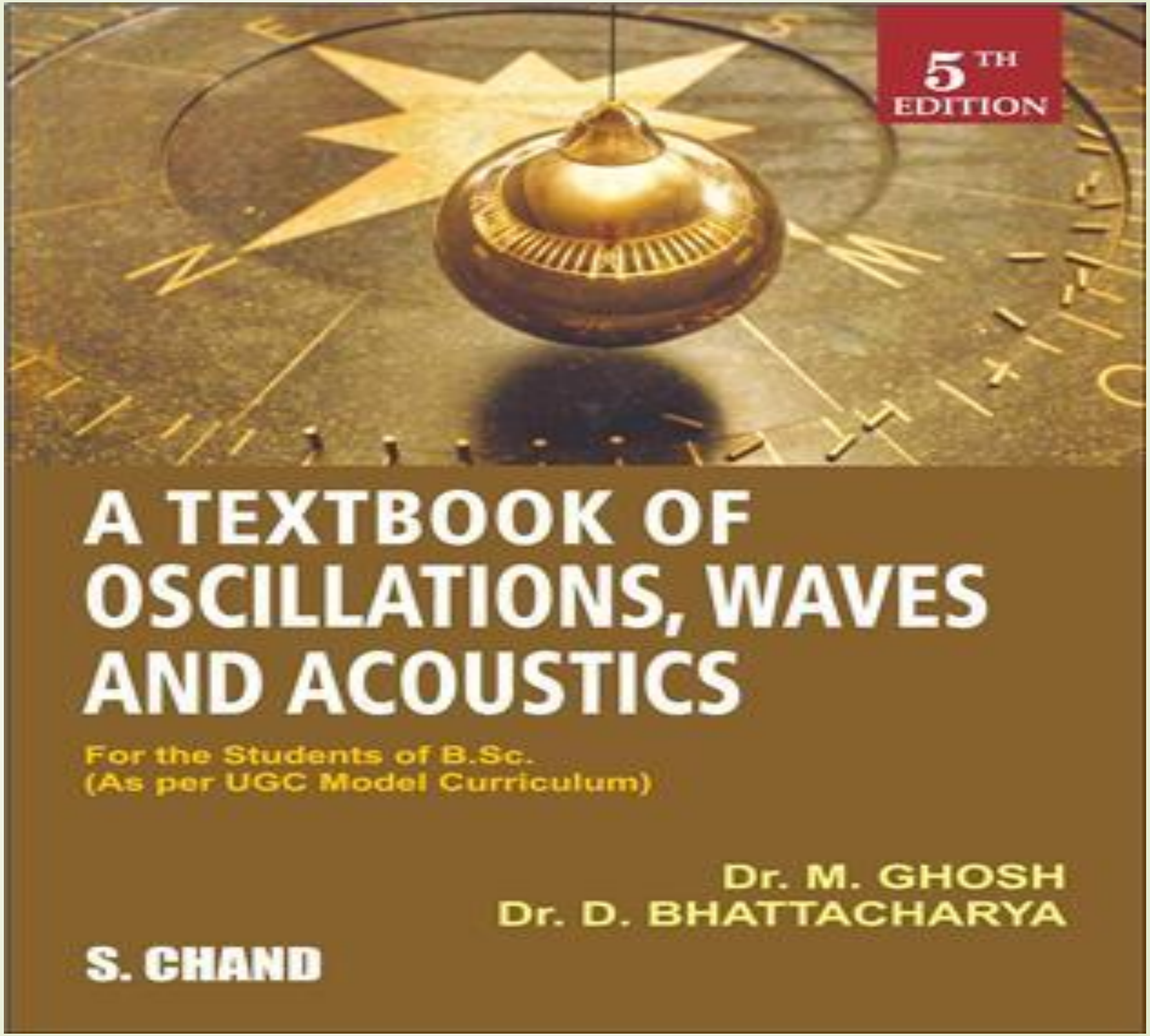

5TH EDITION
A TEXTBOOK OF OSCILLATIONS, WAVES AND ACOUSTICS
For the Students of B.Sc.
(As per UGC Model Curriculum)
Dr. M. GHOSH
Dr. D. BHATTACHARYA
S. CHAND

❖ **Wave**:

A wave is a disturbance in a medium that carries energy without a net movement of particles. It may take the form of elastic deformation, a variation of pressure, electric or magnetic intensity, electric potential, or temperature.

❑ **What are the types of waves?**

The following are the types of waves:

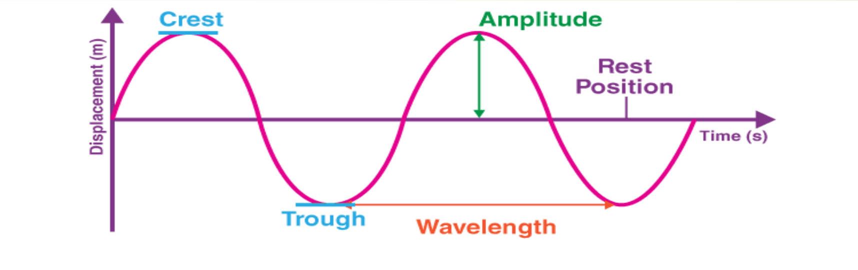
1. Mechanical waves

2.Electromagnetic waves

3. Matter waves

1.Mechanical Wave

A mechanical wave is a wave that is an oscillation of matter and is responsible for the transfer of energy through a medium.



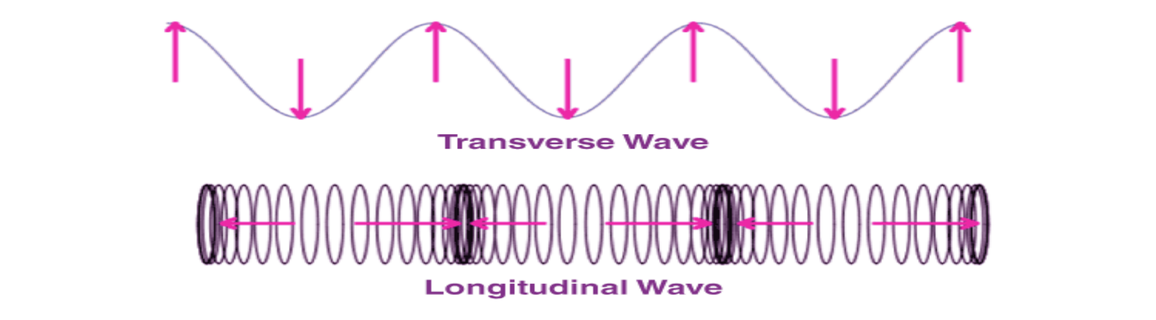
❖ **There are two types of mechanical waves:**

1. Transverse waves :

When the movement of the particles is at right angles or perpendicular to the motion of the energy, then this type of wave is known as a transverse wave. Light is an example of a transverse wave.

2. Longitudinal waves :

In this type of wave, the movement of the particles is parallel to the motion of the energy, i.e. the displacement of the medium is in the same direction in which the wave is moving. Example – Sound Waves, Pressure Waves



- **Electromagnetic Waves:**

These waves are disturbance that does not need any object medium for propagation and can easily travel through the vacuum. They are produced due to various magnetic and electric fields. The periodic changes that take place in magnetic and electric fields and therefore known as electromagnetic waves.

Following are the different types of electromagnetic waves:

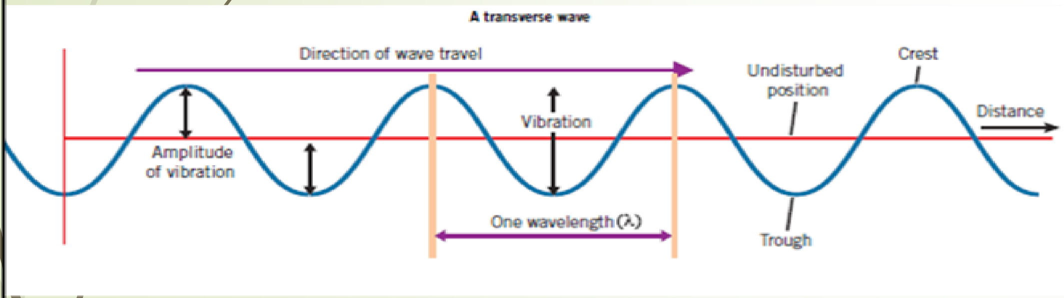
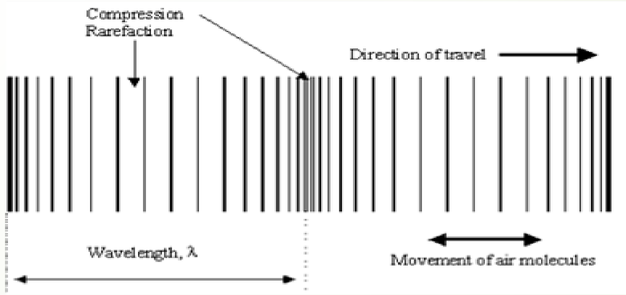
1. Microwaves

2. X-ray

3. Radio waves

4. Ultraviolet waves

❖ Difference Between Transverse Wave & longitudinal Wave

Transverse	**Longitudinal**
Oscillations perpendicular to direction of travel	Oscillations parallel to the direction of travel
Can travel through a vacuum – doesn't need particles to transfer energy	Can't travel through a vacuum – needs particles to transfer energy
Transports energy without transporting matter	Transports energy without transporting matter
e.g. Electromagnetic waves	e.g. Sound waves
	

❖ Matter Waves:

Any moving object can be described as a wave When a stone is dropped into a pond, the water is disturbed from its equilibrium positions as the wave passes; it returns to its equilibrium position after the wave has passed.

❖ Difference Between Mechanical Wave and Non-Mechanical Waves

Mechanical Waves vs Electromagnetic Waves	
Mechanical Wave	Electromagnetic Wave
Mechanical waves are waves that need a medium for propagation.	Non-mechanical waves are waves that do not need any medium for propagation.
Sound waves, water waves and seismic waves are some examples of mechanical waves.	The electromagnetic wave is the only non-mechanical wave.
Mechanical waves cannot travel through vacuum	Non-mechanical waves can travel through vacuum

❖ Doppler Effect

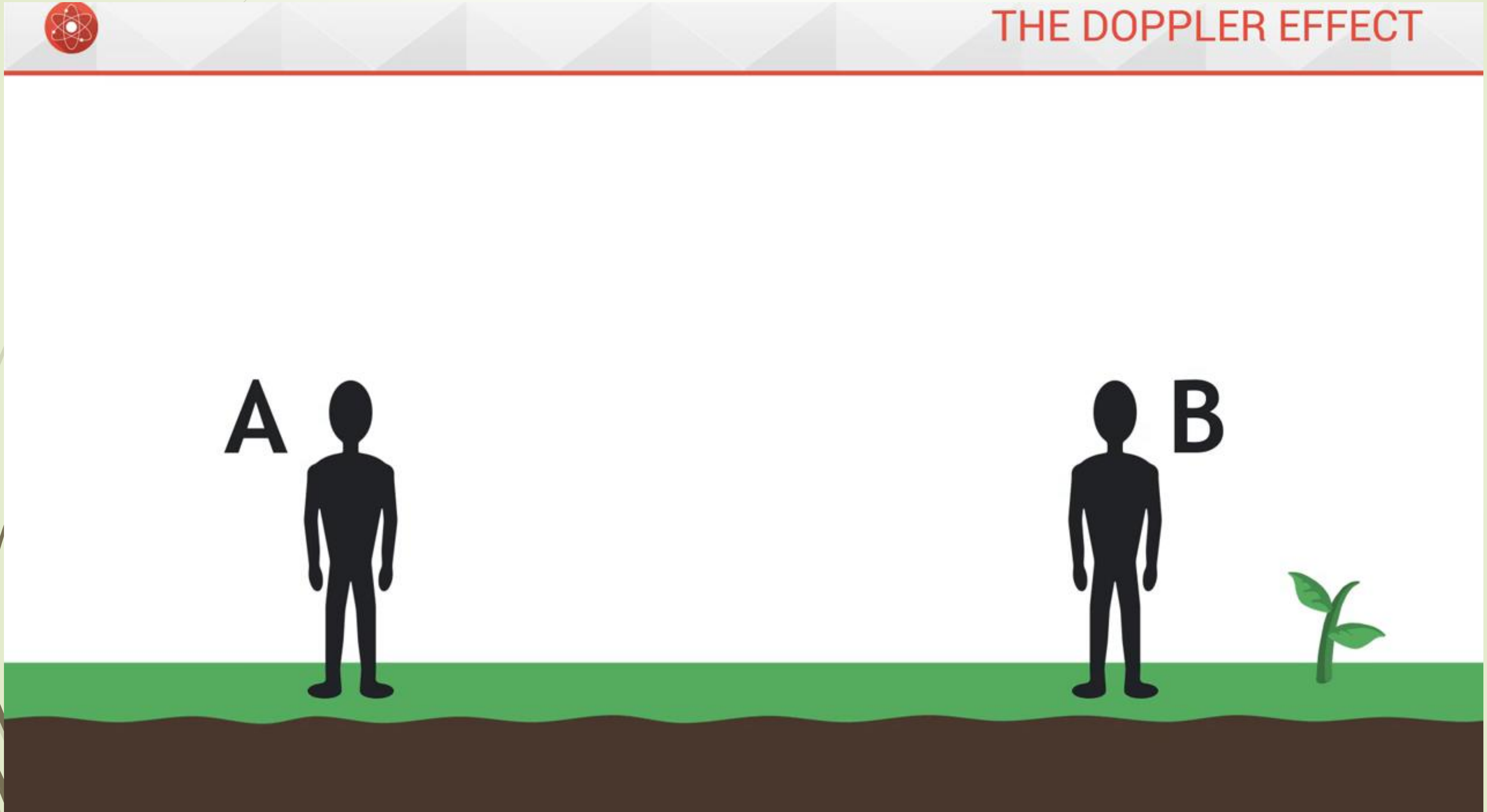
The Doppler effect is the apparent change in the frequency of sound, light, or other waves due to the relative motion between the source of the sound and the observer.

We can deduce the apparent frequency in the Doppler effect using the following equation:

$$f' = \frac{(V \pm V_0)}{(V \pm V_s)} f$$

f' = observed frequency
f = actual frequency
V = velocity of sound waves
V_0 = velocity of observer
V_s = velocity of the source

Real-Life Applications of the Doppler Effect



(a) Source Moving Towards the Observer at Rest

In this case, the observer's velocity is zero, so V_0 is equal to zero. Substituting this into the Doppler effect equation above, we get the equation of the Doppler effect when a source is moving towards an observer at rest as:

$$f' = \frac{V}{(V - V_s)} f$$

f' = observed frequency
f = actual frequency
V = velocity of sound waves
V_s = velocity of the source

(b) Source Moving Away from the Observer at Rest

Since the velocity of the observer is zero, we can eliminate V_0 from the equation. But this time, the source moves away from the observer, so its velocity is negative to indicate the direction. Hence, the equation now becomes as follows:

$$f' = \frac{V}{(V - (-V_s))} f$$

f' = observed frequency
f = actual frequency
V = velocity of sound waves
V_s = velocity of the source

(c) Observer Moving Towards a Stationary Source

In this case, v_s will equal to zero, hence we get the following equation

$$f' = \frac{(V + V_0)}{V} f$$

f' = observed frequency

f = actual frequency

V = velocity of sound waves

V_0 = velocity of observer

(d) Observer Moving Away from a Stationary Source

Since the observer is moving away, the velocity of the observer becomes negative. So, instead of adding V_0, we now subtract, since V_0 is negative.

$$f' = \frac{(V - V_0)}{V} f$$

f' = observed frequency

f = actual frequency

V = velocity of sound waves

V_0 = velocity of observer

Doppler Effect Solved Problems

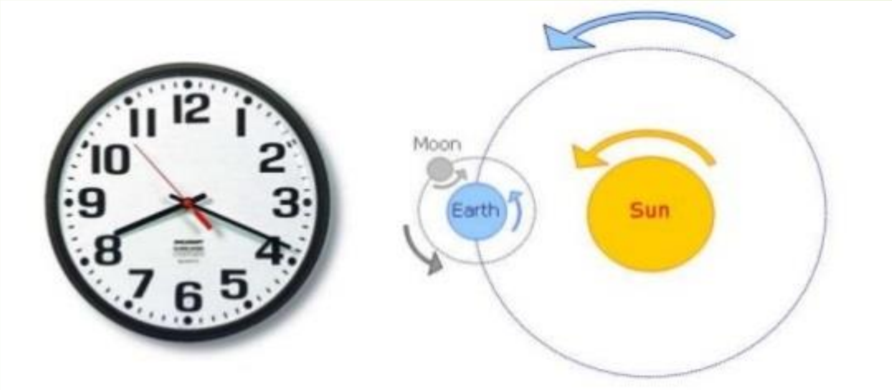
1. Two trains A and B are moving toward each other at a speed of 432 km/h. If the frequency of the whistle emitted by A is 800 Hz, then what is the apparent frequency of the whistle heard by the passenger sitting in train B. (The velocity of sound in air is 360 m/s).

2. A bike rider approaching a vertical wall observes that the frequency of his bike horn changes from 440 Hz to 480 Hz when it gets reflected from the wall. Find the speed of the bike if the speed of sound is 330 m/s.

Ans= The speed of the bike is 14.3 m/s

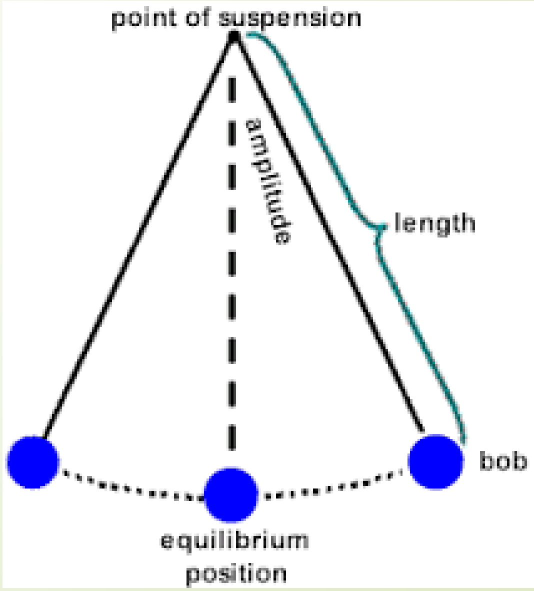
Periodic Motion

A motion that repeats itself repeatedly after a regular interval of time is known as periodic motion. This path may be circular, or elliptical. Linear or more complex.



Oscillatory Motion

A particle having periodic motion remains half of its time period in one direction and the rest of time period remains in another direction along the same line, then its motion is called oscillatory motion

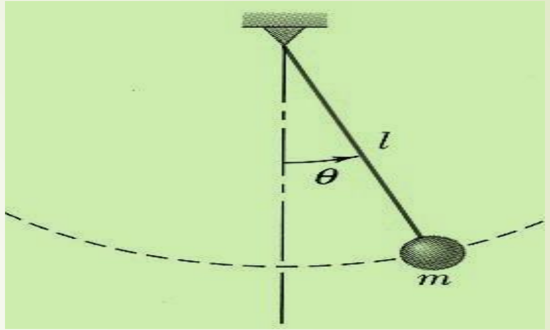


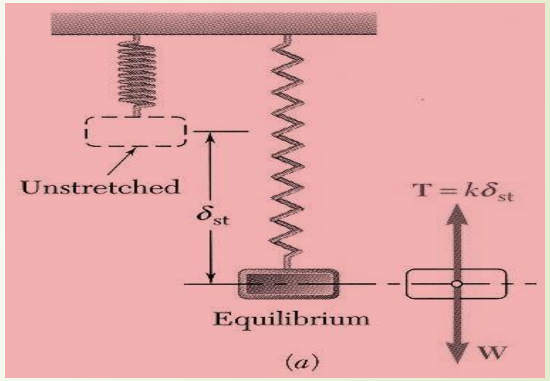
Oscillations and Vibrations:

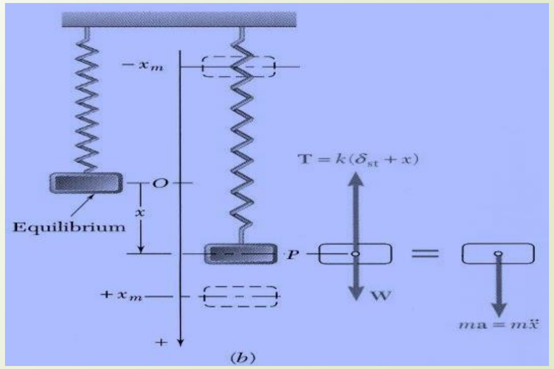
If you pull a swing or pendulum to the side and release it, it oscillates (Latin for "swing") back and forth. If you hang a weight on a spring, pull it down and release it, then the system will vibrate (Latin for "shake") up and down. Oscillations and vibrations are two words for one concept, i.e. repetitive motion.

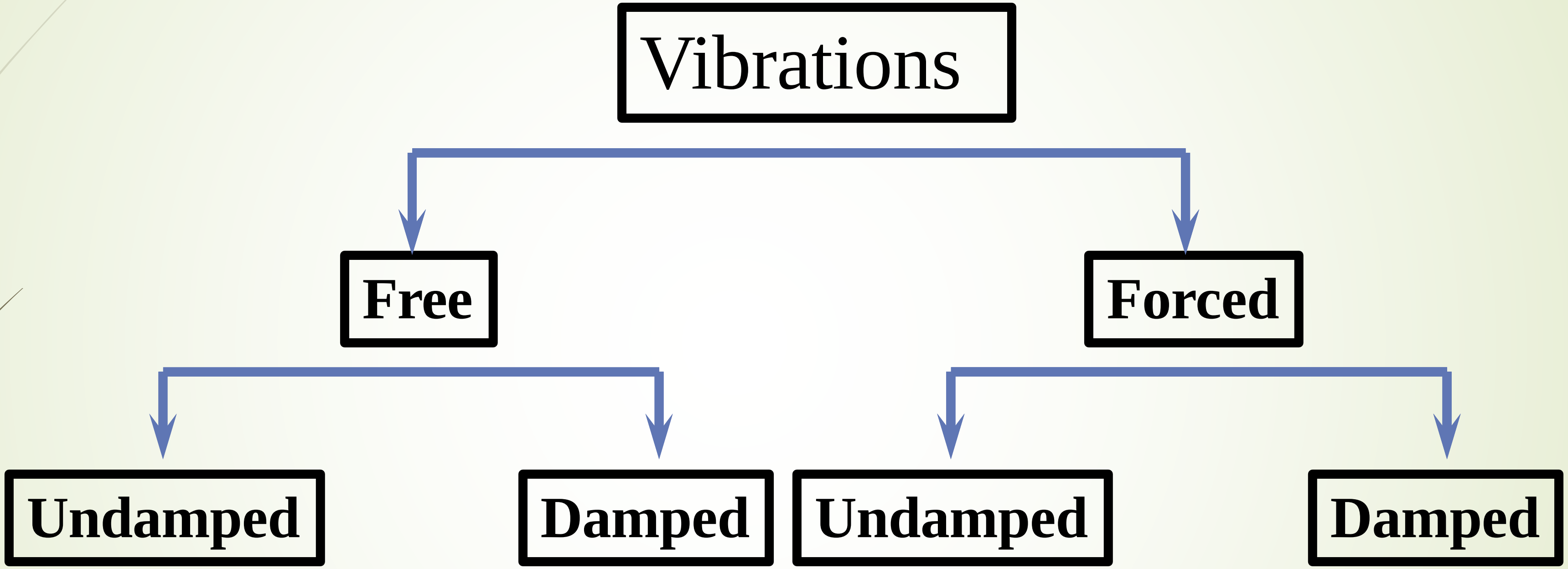
When the time between repetitions is constant, the oscillation is called a harmonic motion and the time between repetitions is called the period. The number of repetitions per second is called the frequency and is the inverse of the period.

The period, T, is normally measured in seconds, and the frequency, ν, in Hertz. When the oscillations (or vibrations) affect the material around them a wave is formed which transports energy away.







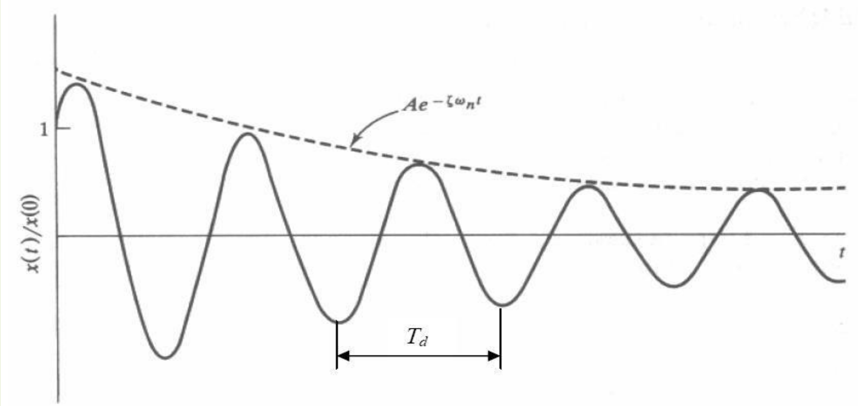

Vibrations
Free
Forced
Undamped
Damped
Undamped
Damped

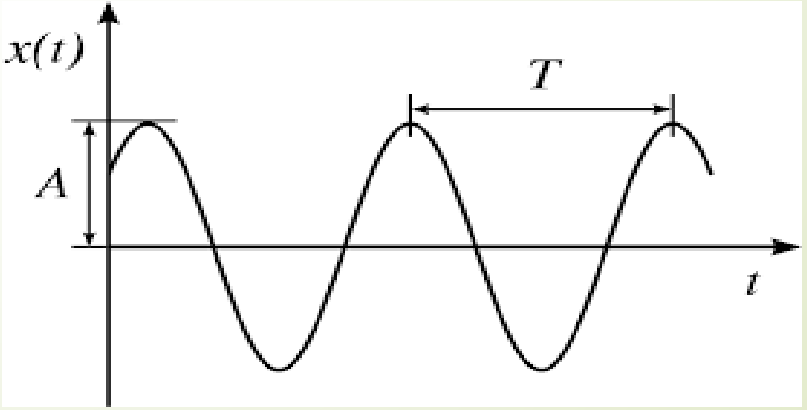
Damped and Forced Vibration

- Maximum displacement of the system from the equilibrium position is the ***amplitude*** of the vibration.

- When the motion is maintained by the restoring forces only, the vibration is described as ***free vibration.***
- When a periodic force is applied to the system, the motion is described as ***forced vibration.***
- When the frictional dissipation of energy is neglected, the motion is said to be ***undamped.***
- **Actually, all vibrations are *damped* to some degree.**

Damped vibration: When a vibrating body vibrates in air or any other resisting medium, the amplitude of vibration does not remain constant but decreases gradually and ultimately the body comes to rest. Such vibrations are known as damped vibrations.





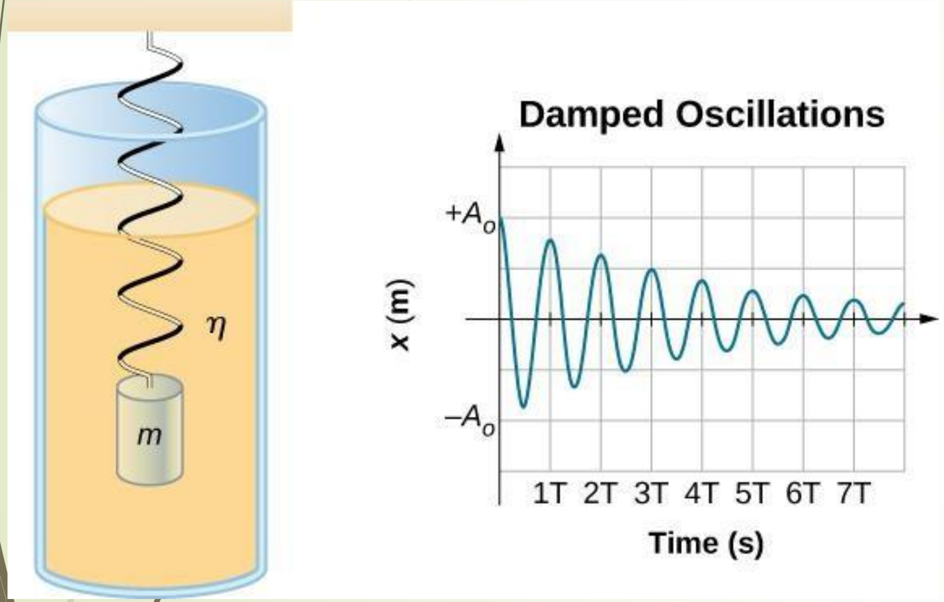
Free Oscillation and Damped Oscillation

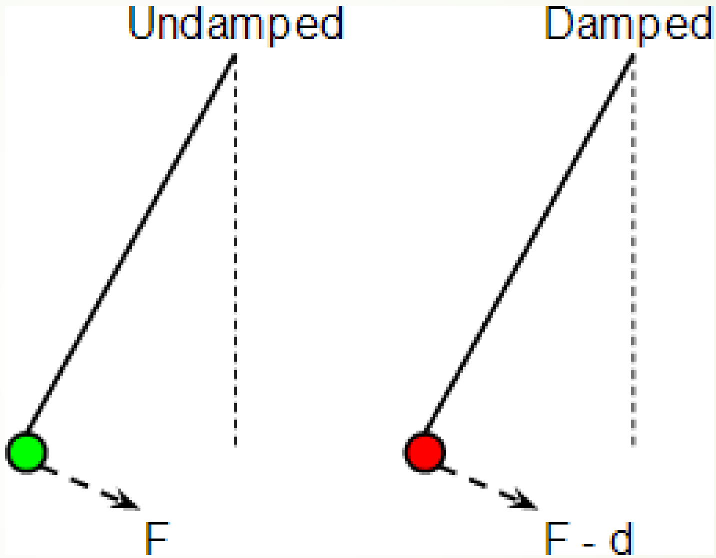
- If an oscillation occurs flawlessly without any resistive force acting on it is called free oscillation.
- Any oscillation occurring in an air medium, experiences frictional force and consequent energy dissipation occurs.
- The amplitude of oscillation decays continuously with time and finally diminishes. Such oscillation is called damped oscillation.
- The dissipated energy appears as heat either within the oscillating system itself or in the surrounding medium.

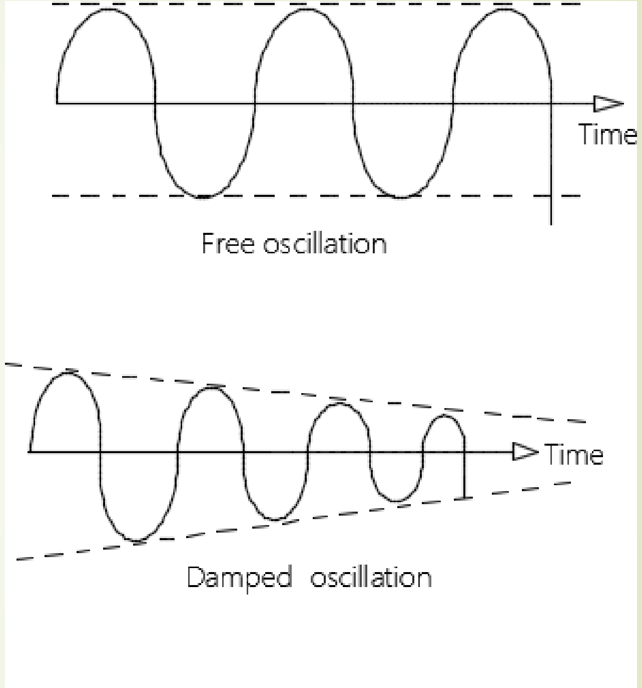
Characteristics of Damped Oscillation

- Frictional force acting on a body opposite to the direction of its motion is called damping force.
- Damping force reduces the velocity and the kinetic energy of the moving body.
- Damping or dissipative forces generally arises due to the viscosity or friction in the medium and are non-conservative in nature.
- When velocities of body are not high, damping force is found to be proportional to velocity (v) of the particle
- The frequency of damped oscillator is always less than that of it's natural or undamped frequency.
- Amplitude of oscillation does not remain constant, rather it decays with time

Free Oscillation and Damped Oscillation







Free and damped oscillations

Time-30 **Marks-15**

1. Explain the Doppler effect.

What is the apparent frequency of the whistle heard by the passenger sitting in train B when two trains A and B are moving toward each other at a speed of 432 km/h? If the frequency of the whistle emitted by A is 800 Hz, then (The velocity of sound in air is 360 m/s).

2. Determine the energy of the simple harmonic oscillator

A 600 g block connected to a spring for which a force constant is 10 N/m is free to oscillate. Determine the period of its motion

Differential Equation of a Damped Oscillator

If damping is taken into consideration for an oscillator, then oscillator experiences

(i) Restoring Force : $F_r = -ky$; k=force constant

(ii) Damping Force : $F_d = -b\frac{dy}{dt}$; b=damping constant

Where, y is the displacement of oscillating system and v is the velocity of this displacement.

We, therefore, can write the equation of the damped harmonic oscillator as, $F = F_d + F_r$

From Newton's 2nd law of motion, $F = m\frac{d^2y}{dt^2}$

Combination of Hook's law and Newton's 2nd law of motion:

$$m\frac{d^2y}{dt^2} = -ky - b\frac{dy}{dt}$$

$$\Rightarrow \frac{d^2y}{dt^2} + \frac{k}{m}y + \frac{b}{m}\frac{dy}{dt} = 0$$

$$\Rightarrow \frac{d^2y}{dt^2} + 2p\frac{dy}{dt} + \omega^2 y = 0 \qquad (4.1)$$

$2p = \frac{b}{m}$ = damping co-efficient of the medium.

p has the dimension of frequency referred to as damping frequency.

Solution:

To solve equation (4.1) let us take the trial solution,

$$y = Ae^{m't} \qquad (4.2)$$

Substituting this solution in equation (4.1) we get,

$$m'^2Ae^{m't} + 2pm'Ae^{m't} + \omega^2Ae^{m't} = 0$$

$$\Rightarrow m'^2y + 2pm'y + \omega^2y = 0$$

$\Rightarrow m'^2 + 2pm' + \omega^2 = 0$; [Quadratic equation]

Solving this equation for m' we get,

$$m' = -\frac{2p \pm \sqrt{4p^2 - 4\omega^2}}{2} = -p \pm \sqrt{p^2 - \omega^2}$$

Then, the general solution of equation (4.1) is,

$$y = e^{-pt}\left[Ae^{\left(\sqrt{p^2-\omega^2}\right)t} + Be^{-\left(\sqrt{p^2-\omega^2}\right)t}\right] \qquad (4.3)$$

Case. I (Overdamped motion)

If $p^2>\omega^2$,the indices of *"e"* are real and we get,

$$y = e^{-pt}[Ae^{\alpha t} + Be^{-\alpha t}] \qquad (4.4)$$

Where, $\alpha = \sqrt{p^2-\omega^2}$

Now, let us replace A and B by two other constants C and δ such that we can write, $A=\frac{C}{2}e^{\delta}$ and $B=\frac{C}{2}e^{-\delta}$

Here, $A+B=\frac{C}{2}e^{\delta} + \frac{C}{2}e^{-\delta} = \frac{C}{2}\left(e^{\delta} + e^{-\delta}\right) = \frac{C}{2}2\cosh\delta$

$\therefore A + B = C\cosh\delta$

$$\frac{A}{B} = \frac{\frac{C}{2}e^{\delta}}{\frac{C}{2}e^{-\delta}} = e^{2\delta}$$

Using the new constants in equation (4.4),

$$y = e^{-pt}\left[\frac{C}{2}e^{\delta}e^{\alpha t} + \frac{C}{2}e^{-\delta}e^{-\alpha t}\right]$$

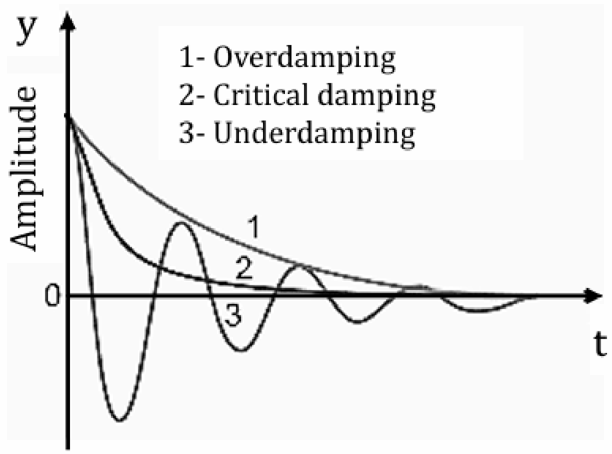
$$=\frac{C}{2}e^{-pt}\left[e^{(\alpha t+\delta)} + e^{-(\alpha t+\delta)}\right]$$

$$=\frac{C}{2}e^{-pt} \times 2\cosh(\alpha t+\delta)$$

$$=Ce^{-pt}\cosh(\alpha t+\delta)$$

So, $y=Ce^{-pt}\cosh\left[\left(\sqrt{p^2-\omega^2}\,t\right)+\delta\right] \qquad (4.5)$

Negative power of *"e"* indicates exponential decrease of y that means the particle does not oscillate. Equation (4.5) represents a continuous return of y from its maximum value to zero at t=∞ without oscillation. This type of motion is called the overdamped or dead beat or aperiodic motion.



Example:

Dead beat galvanometer, pendulum oscillating in a viscous fluid, etc.

Then, the general solution of equation (4.1) is,

$$y = e^{-pt}\left[Ae^{\left(\sqrt{p^2-\omega^2}\right)t} + Be^{-\left(\sqrt{p^2-\omega^2}\right)t}\right] \qquad (4.3)$$

Case. II (Underdamped motion)

If $p^2<\omega^2$,the indices of "*e*" are imaginary and we get,

Where, $\theta=\sqrt{(\omega^2 - p^2)}$

$$y = e^{-pt}\left[Ae^{i\theta t} + Be^{-i\theta t}\right]$$

$$=e^{-pt}[Acos\theta t + iAsin\theta t + Bcos\theta t - iBsin\theta t]$$

$$=e^{-pt}[(A + B)cos\theta t + i(A - B)sin\theta t] \qquad (4.5)$$

Let, $(A+B)=a\,cos\gamma$ and $i(A\text{-}B)=a\,sin\gamma$

$$a = \sqrt{a^2cos^2\gamma + a^2sin^2\gamma} = \sqrt{(A + B)^2 + i^2(A - B)^2}$$

$$=\sqrt{A^2 + 2AB + B^2 - A^2 + 2AB - B^2} = \pm 2\sqrt{AB}$$

$$\tan\gamma=\frac{asin\gamma}{acos\gamma}=\frac{i(A-B)}{(A+B)}$$

Using the new constants in equation (4.5),

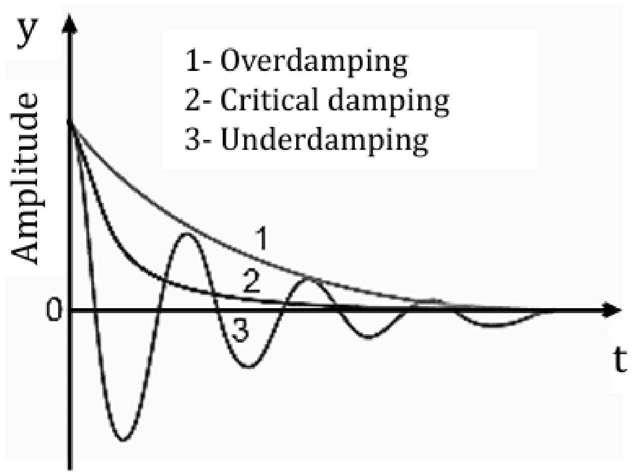
$$y = e^{-pt}[acos\gamma\, cos\theta t + asin\gamma\, sin\theta t]$$

$$y=ae^{-pt}[cos\theta t\,cos\gamma + sin\theta t\,sin\gamma]$$

$$=ae^{-pt}cos(\theta t-\gamma)$$

$$y=ae^{-pt}cos\left[\sqrt{(\omega^2 - p^2)}t-\gamma\right] \qquad (4.6)$$

In this case y alternates in sign and we have periodic motion but the amplitude continuously diminishes due to the factor e^{-pt}. This situation is called underdamping with the amplitude ae^{-pt} and the frequency $\sqrt{(\omega^2 - p^2)}$.



Then, the general solution of equation (4.1) is,

$$y = e^{-pt}\left[Ae^{\left(\sqrt{p^2-\omega^2}\right)t} + Be^{-\left(\sqrt{p^2-\omega^2}\right)t}\right] \qquad (4.3)$$

Case. III (Critical damping motion)

If $p^2=\omega^2$, $(p^2-\omega^2)=0$; So, $p^2=\omega^2, p=\omega$

From equation (4.3) we can write,

$$y = e^{-\omega t}[Ae^0 + Be^0]$$

$$=e^{-\omega t}[A+B]$$

It implies that the oscillation is decaying without any damping factor. It is not possible. So, the solution breaks down. Now, we have to consider that p^2 is not quite equal to ω^2, but very close to each other. Thus $\sqrt{p^2-\omega^2} = h \approx 0$ (close to zero but not zero).

From equation (Using the new constants in equation (4.3),

$$y = e^{-pt}[Ae^{ht} + Be^{-ht}] = e^{-pt}\left[A\left(1+ht+\frac{h^2t^2}{2!}+\frac{h^3t^3}{3!}+\cdots\right)+ B\left(1-ht+\frac{h^2t^2}{2!}-\frac{h^3t^3}{3!}+\cdots\right)\right] = e^{-pt}[A(1+ht)]+B(1-ht)]$$

$$y=e^{-pt}[(A+B)+(A-B)ht] \qquad (4.7)$$

Let, $A+B=A'$ and $(A-B)h=B'$

$$y=e^{-pt}[A'+B't] \qquad (4.8)$$

At amplitude, $y=y_{max}=a$ (at $t=0$)

Applying these two conditions in equation (4.8),

$$a=e^0(A'+B'\times 0) \Rightarrow A'=a$$

$$\frac{dy}{dt} = -pe^{-pt}(A'+B't) + e^{-pt}B'$$

$$\left[\frac{dy}{dt}\right]_{t=0} = -pe^0(A'+B'\times 0) + e^0B' = 0$$

$$\Rightarrow -pA' + B' = 0$$

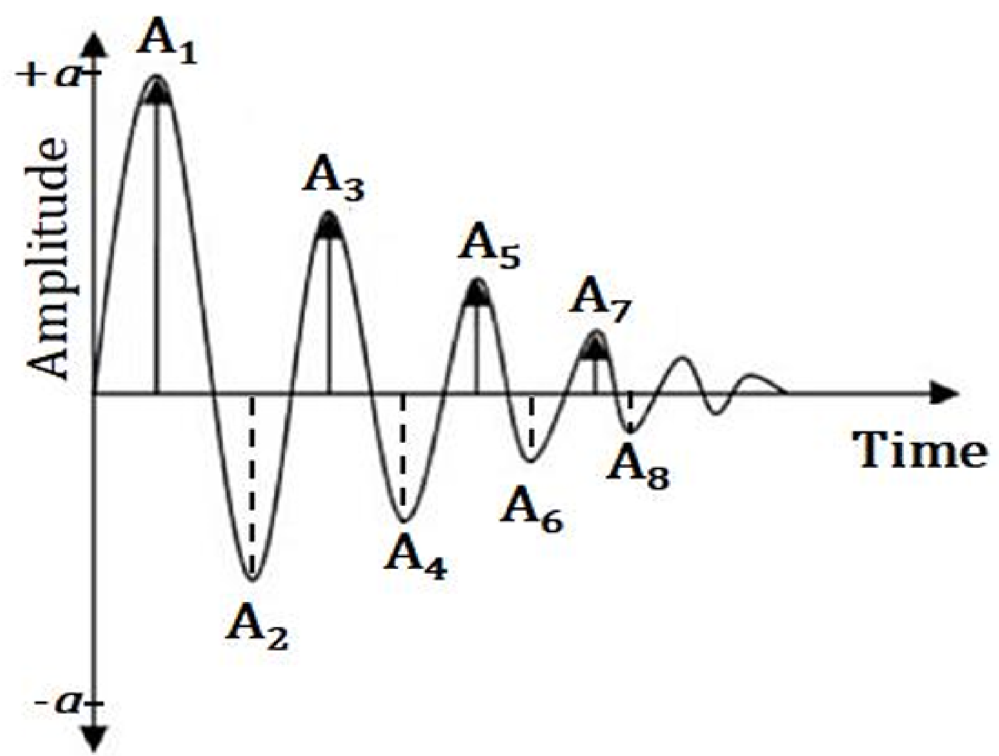
$$\Rightarrow B' = pa$$

So, from equation (4.8)

$$y = e^{-pt}[a+pat]$$

$$y = ae^{-pt}[1+pt] \qquad (4.9)$$

This solution represents a continuous return of y from its amplitude to zero. Although it looks like overdamped motion it is a boundary between underdamped and overdamped motion. Under this condition oscillatory motion changes over to dead beat motion and vice versa. Hence, this is called critical damping motion.



- Angular frequency of a damped oscillator, $\omega' = \sqrt{\omega^2 - p^2}$
- Since, $\omega^2 = \frac{k}{m}$ and $2p = \frac{b}{m}$; $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$
- Mechanical energy of a free oscillator, $E = \frac{1}{2}ka^2 =$constant
- Mechanical energy of a damped oscillator, $E = \frac{1}{2}ka^2\, e^{-2pt} = \frac{1}{2}ka^2\, e^{-\frac{b}{m}t}$; [reduces with exponentially with time]

Damped vibration

During the motion of a body executing damped simple harmonic vibration, the following two froces will be simultaneously acting on the body.

(a) the restoring force acting on the body which is proportional to the displacement and directed opposite to the displacement. That is, $F \propto -ay$.

(b) the damping force proportional to the velocity and directed opposite to the velocity. Let this force be $-b\frac{dy}{dt}$.

$$F_{net} = F_r + F_d$$

$$\therefore m\frac{d^2y}{dt^2} = -ay - b\frac{dy}{dt}$$

Damped vibration

Therefore the differential equation of motion shall be as follows ; $m \frac{d^2y}{dt^2} = -b \frac{dy}{dt} - ay$

$$\text{or,} \quad m \frac{d^2y}{dt^2} + b \frac{dy}{dt} + ay = 0$$

$$\text{or,} \quad \frac{d^2y}{dt^2} + \frac{b}{m} \frac{dy}{dt} + \frac{a}{m} y = 0$$

$$\text{or,} \quad \frac{d^2y}{dt^2} + \lambda \frac{dy}{dt} + \mu y = 0 \quad \cdots \quad \cdots \quad (1)$$

where $\lambda = \frac{b}{m}$ and $\mu = \frac{a}{m}$.

Equation (2.1) may be used in order to deduce the way in which y varies with t and thus predict the nature of the motion, i.e., whether the motion is oscillatory or dead-beat.

Damped vibration

(i) If $\lambda = 0$ i.e., if there is no frictional force either interal or external, then equation **(1)** reduces to

$$\frac{d^2y}{dt^2} + \mu y = 0$$

or, $$\frac{d^2y}{dt^2} = -\mu y$$

i. e., acceleration $\propto$ displacement and opposed to it.

$\therefore$ the equation represents an undamped simple harmonic vibration of period T given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\mu}}$$

and the frequency is given by $n = \frac{1}{T} = \frac{\sqrt{\mu}}{2\pi}$

The period T and the frequency n in the above equation are known as natural period and natural frequency of vibration of the body.

Damped vibration

by m and putting $\frac{b}{m} = 2k$ and $\frac{a}{m} = \omega_0^2$ we get

$$\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + \omega_0^2 y = 0 \qquad (3)$$

Let a solution of differential equation be

$$y = Ae^{\alpha t}$$

$$\therefore \quad \frac{dy}{dt} = A\alpha e^{\alpha t} \text{ and } \frac{d^2y}{dt^2} = A\alpha^2 e^{\alpha t} \qquad (3)$$

Damped vibration

Substituting these values in (3) we have

$$A\alpha^2 e^{\alpha t} + 2kA\alpha e^{\alpha t} + \omega_0^2 A e^{\alpha t} = 0$$

$$\alpha^2 + 2k\alpha + \omega_0^2 = 0$$

$$\therefore \quad \alpha = \frac{-2k \pm \sqrt{4k^2 - 4\omega_0^2}}{2}$$

$$= -k \pm \sqrt{k^2 - \omega_0^2}$$

Hence a general solution of the differential equation (3) is

$$y = A_1 e^{(-k + \sqrt{k^2 - \omega_0^2})\,t} + A_2 e^{(-k - \sqrt{k^2 - \omega_0^2})\,t} \quad ..$$

where A_1 and A_2 are arbitrary constants whose values are determined from boundary conditions.

The above equation can be put in the form

$$y = e^{-kt}\left[A_1 e^{\sqrt{k^2 - \omega_0^2}\,t} + A_2 e^{-\sqrt{k^2 - \omega_0^2}\,t}\right] \quad (4)$$

The nature of motion represented by equation (4) depends upon the relative values of k and ω_0. Now, three different cases arise.

Damped vibration

(i) **Heavy damping.** When $k > \omega_0$. In this case $k^2 - \omega_0^2$

$= \frac{r^2}{4m^2} - \frac{s}{m}$ = *a positive* quantity. Hence $\sqrt{k^2 - \omega_0^2}$ is real.

Substituting $\sqrt{k^2 - \omega_0^2} = p$ in equation (2.3) we have

$$y = e^{-kt}\left[A_1 e^{pt} + A_2 e^{-pt}\right]$$

Let $A = A_1 + A_2$ and $B = A_1 - A_2$ then

$$A_1 = \frac{A+B}{2} \quad \text{and} \quad A_2 = \frac{A-B}{2}$$

$$\therefore \quad y = e^{-kt}\left[\frac{A+B}{2} e^{pt} + \frac{A-B}{2} e^{-pt}\right]$$

$$= e^{-kt}\left[A\,\frac{e^{pt} + e^{-pt}}{2} + B\,\frac{e^{pt} - e^{-pt}}{2}\right]$$

$$= e^{-kt}\left[A \cosh pt + B \sinh pt\right]$$

The displacement consists of two terms both dying off exponentially to zero.

The volues of A and B depend upon the initial condition. If $y = 0$ at $t = 0$. Then

$$A = 0 \text{ and } \quad y = Be^{-pt} \sinh pt$$

$$= B e^{-pt} \sinh \sqrt{(k^2 - \omega_0^2)}\; t$$

Damped vibration

(*ii*) **Critical damping when $k=\omega_0$.** In this case

$$k^2-\omega_0^2=\frac{r^2}{4m^2}-\frac{s}{m}=0$$

and the quadratic equation in α has two *equal* roots

The displacement y is given by

$$y=(A_1+A_2)e^{-kt}$$

This is the limiting case of the behaviour shown in (*i*). We shall see in the next case when $k<\omega_0$ or $k^2-\omega_0^2$ is a negative quantity that the particle undergoes an oscillatory damped simple harmonic motion. Therefore, for $k=\omega_0$ the motion is neither over-damped nor oscillatory and is said to be *critically damped*. The property of critical damping is made use of in measuring instruments like ballistic galvanometers.

Damped vibration

(*iii*) **Light damping when** $k < \omega_0$. In this case $k^2 - \omega_0^2$ $= \frac{r^2}{4m^2} - \frac{s}{m} =$ *a negative quantity*. Hence $\sqrt{k^2 - \omega_0^2}$ is an *imaginary quantity*.

Let $\sqrt{k^2 - \omega_0^2} = i\omega'$ or $\omega' = \sqrt{\omega_0^2 - k^2}$

$$\therefore \quad y = e^{-kt}[A_1 e^{i\omega' t} + A_2 e^{-i\omega' t}]$$

Put $A_1 = \frac{A_0}{2i} e^{i\phi}$ and $A_2 = \frac{A_0}{2i} e^{-i\phi}$

where A_0 and ϕ are also constants the value of which depends upon the state of motion at $t = 0$

$$\therefore \quad y = e^{-kt} A_0 \left[\frac{e^{i(\omega' t + \phi)} - e^{-i(\omega' t + \phi)}}{2i} \right]$$

$$= A_0 e^{-kt} \sin(\omega' t + \phi) \qquad \ldots(v)$$

This is the equation of a *damped simple harmonic* motion with amplitude $A_0 e^{-kt}$ which goes on decreasing with time and angular. frequency $\omega' = \sqrt{\omega_0^2 - k^2}$.

A **simple pendulum** is constructed by attaching a mass to a thin rod or a light string. We will also assume that the amplitude of the oscillations is small.

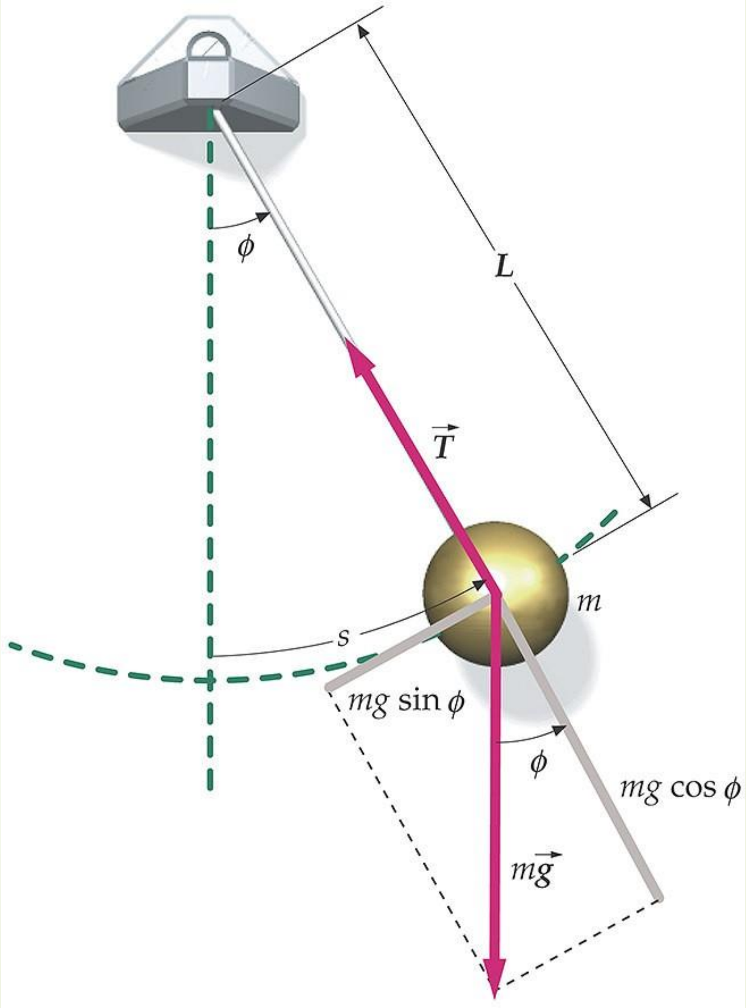
The pendulum is best described using polar coordinates.

The origin is at the pivot point. The coordinates are (r, φ). The r-coordinate points from the origin along the rod. The φ- coordinate is perpendicular to the rod and is positive in the counterclockwise direction.

Apply Newton's 2nd Law to the pendulum

$\sum F_{\varphi}$ = -mg sin $\varphi = ma_{\varphi}$

$\sum F_r$ = T= mg cos φ



If we assume that φ <<1 rad, then sin φ ≈ φ and cos φ ≈1, the angular frequency of oscillations is then:

$$\sum F_{\varphi} = -mg\sin\phi = m\,a_{\varphi}$$

$$\Longrightarrow -mg\sin\phi = m\alpha$$

$$\Longrightarrow -g\sin\phi = \alpha$$

$$\Longrightarrow \alpha = -g\phi$$

$$\Longrightarrow \alpha = -\left(\frac{g}{L}\right)s$$

$$\therefore \alpha = -\omega^2 s \;\;;\omega^2 = \frac{g}{L}$$

This equation represents the SHM

Therefore, The time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$;

Simple Harmonic Motion

Integrating the acceleration gives speed as a function of displacement from the origin. Note the switching of the variable from time ***t*** to velocity ***v*** in the top line.

$$a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{dv}{dx}\cdot v$$

$$a = \frac{dv}{dx}\cdot v = -\left(\frac{k}{m}\right)\cdot x$$

$$v\cdot dv = -\left(\frac{k}{m}\right)\cdot x\cdot dx$$

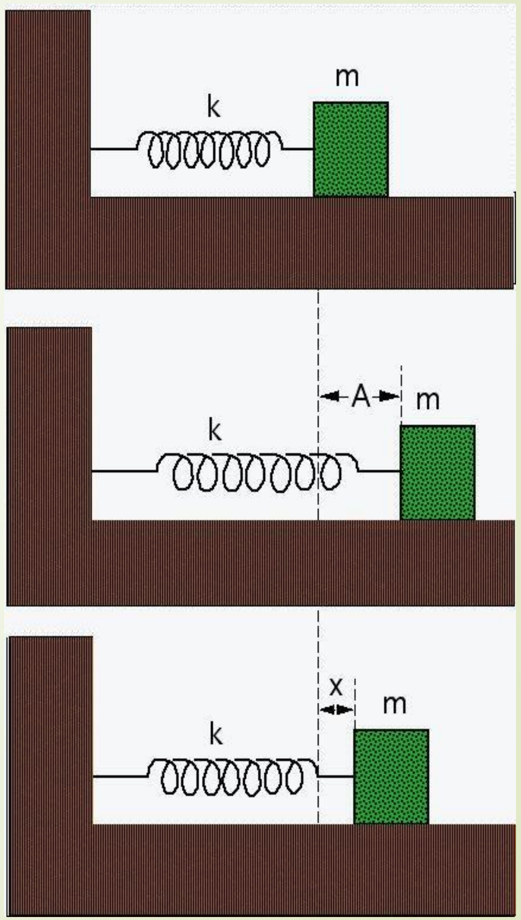
$$\int_0^v v\cdot dv = -\left(\frac{k}{m}\right)\cdot\int_A^x x\cdot dx$$

Putting v=0 when x=A and v=v when x=x

$$\left[\frac{1}{2}v^2\right]_0^v = -\left(\frac{k}{m}\right)\cdot\left[\frac{1}{2}x^2\right]_A^x$$

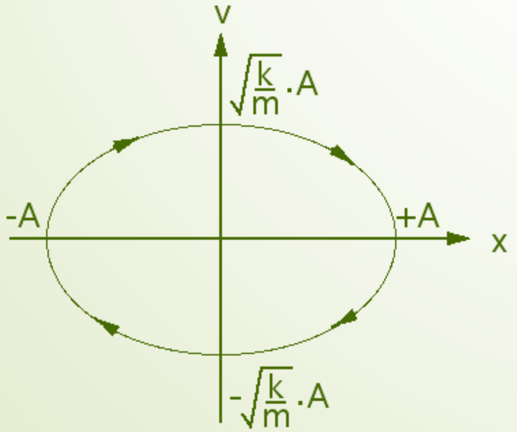
$$v^2 = -\left(\frac{k}{m}\right)\left(x^2 - A^2\right)$$

$$v = \pm\sqrt{\left(\frac{k}{m}\right)\left(A^2 - x^2\right)}$$



A graph of velocity versus displacement is given below.

The speed goes to zero at the extreme displacements, and to maximum at the origin.



Integrating further gives the displacement as a function of time. Note that the variables are separated to give only displacement on the LHS and only time on the RHS.

$$v = \sqrt{\left(\frac{k}{m}\right)\left(A^2 - x^2\right)}$$

$$\frac{dx}{dt} = \sqrt{\left(\frac{k}{m}\right)\left(A^2 - x^2\right)}$$

$$\frac{dx}{\sqrt{\left(A^2 - x^2\right)}} = \sqrt{\frac{k}{m}} \cdot dt$$

now at $t = 0$ the displacement is x_0 so

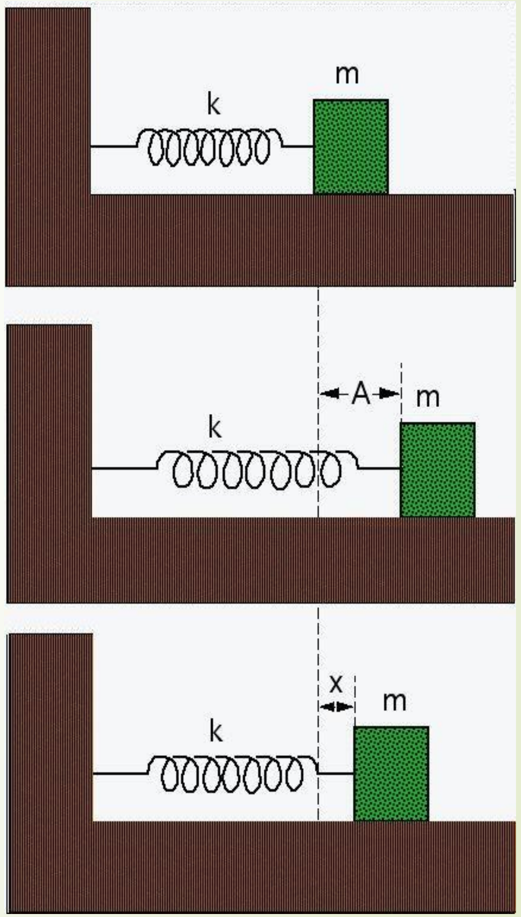
$$\int_{x_0}^{x} \frac{dx}{\sqrt{\left(A^2 - x^2\right)}} = \int_0^t \sqrt{\frac{k}{m}} \cdot dt$$

$$\left[\sin^{-1}\left(\frac{x}{A}\right)\right]_{x_0}^{x} = \sqrt{\frac{k}{m}} \cdot \int_0^t dt$$

$$\sin^{-1}\left(\frac{x}{A}\right) - \sin^{-1}\left(\frac{x_0}{A}\right) = \sqrt{\frac{k}{m}} \cdot t$$

$$\sin^{-1}\left(\frac{x}{A}\right) = \left(\sqrt{\frac{k}{m}} \cdot t + \alpha\right), \text{ where } \sin\alpha = \frac{x_0}{A}$$

$$\text{i.e. } \frac{x}{A} = \sin\left(\sqrt{\frac{k}{m}} \cdot t + \alpha\right)$$



Simple Harmonic Motion

The result is:

$$x = A \cdot \sin(\omega t + \alpha)$$

where $\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$ is the angular frequency

and $\sin\alpha = \frac{x_0}{A}$, gives the initial position

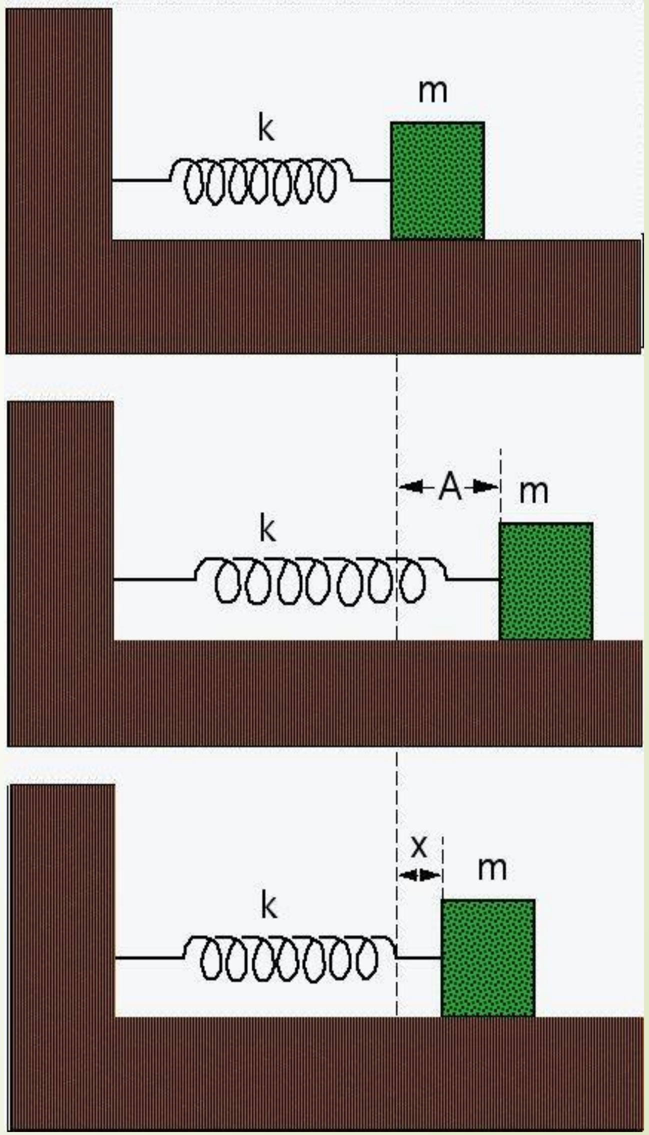
You always have the sine of an *angle*.

Here the angles, α and φ are not real angles in space.

$$\sin\alpha = \frac{x_0}{A} \text{ and}$$

$$\sin(\omega t + \alpha) = \sin\phi = \frac{x}{A}$$

α and φ are called ***phase angles***, because they relate particular displacements to the maximum displacement. ***ωt*** has to have a unit of angle, and so ***ω*** must be in radian per second, rad.s^{-1}.



Projection of Circular Motion

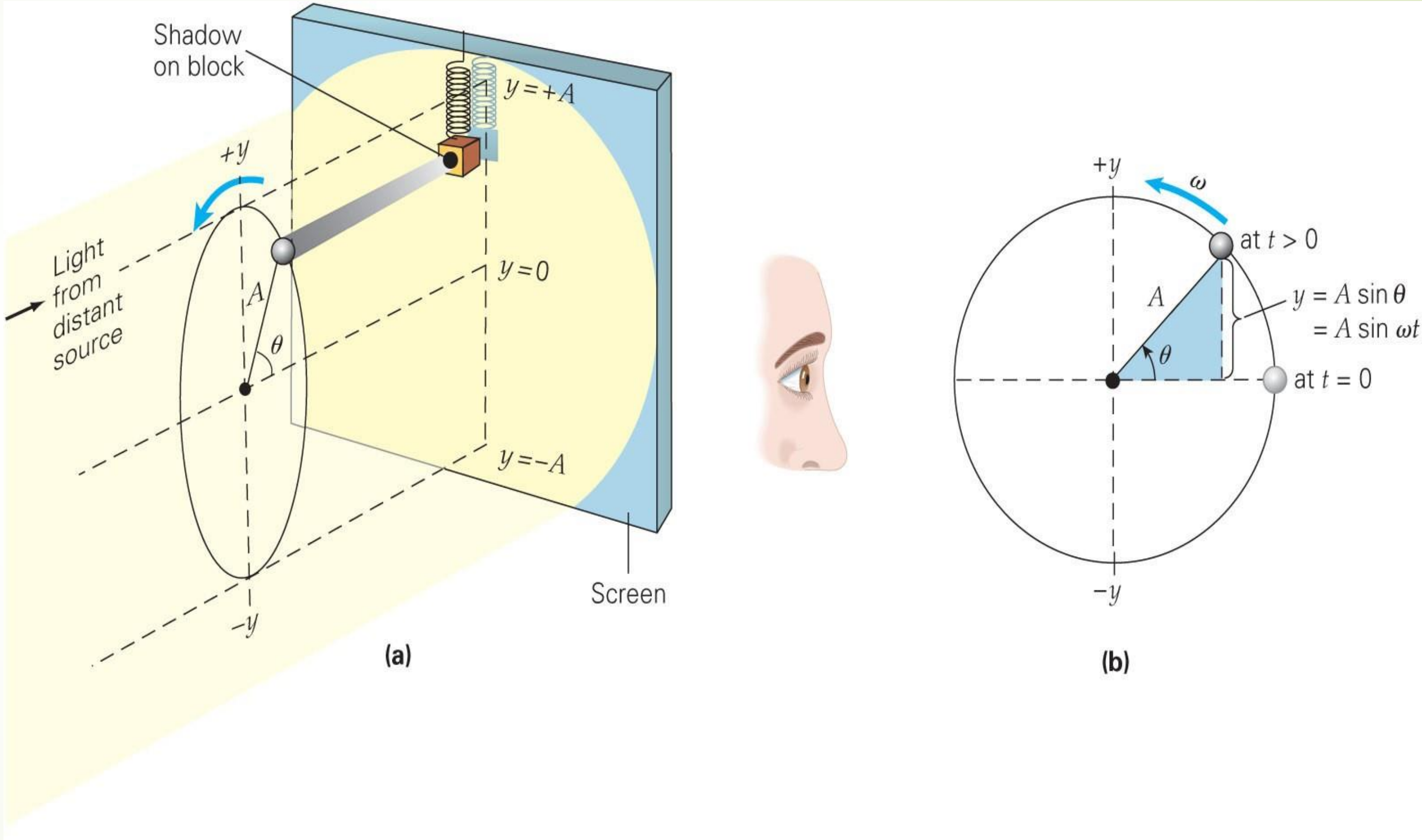
Angular displacement $\theta = \omega t$;

$$\therefore \omega = \frac{\theta}{t} = \frac{2\pi}{T}$$

Therefore, The time period T =

$$\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{k}{m}};$$

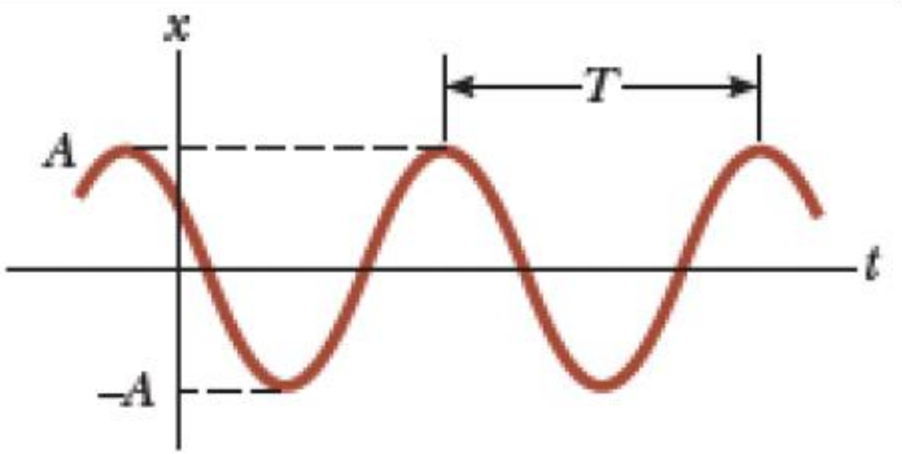
where K is the spring constant and m is the mass , Where T is the time period of the SHM ,ω is the angular velocity



Further investigation of the mathematical description of simple harmonic motion.

$x(t) = A\cos(\omega t + \phi)$

The **angular frequency,** $\boldsymbol{\omega} = \sqrt{\frac{\boldsymbol{k}}{\boldsymbol{m}}}$ and the constant angle ϕ is called the **phase constant** (or initial phase angle) and, along



Simplifying this expression gives $\omega T = 2\pi$ $or\ T = 2\pi/\omega.$

frequency *f* of the motion.

$= 2\pi/T$

$$T = 2\pi/\omega = 2\pi\sqrt{m/k}$$
$$Frequency, f = 1/T = 1/2\pi\sqrt{k/m}$$

$$\boldsymbol{The\ velocity,}\quad \boldsymbol{v = \frac{dx}{dt} = -\omega A sin(\omega t + \phi)}$$

$$\boldsymbol{The\ acceleration,}\quad \boldsymbol{a = \frac{d^2x}{dt^2} = -\omega^2 A cos(\omega t + \phi)}$$

❖ Energy of the Simple Harmonic Oscillator

Simple Harmonic Motion or SHM is defined as a motion in which the restoring force is directly proportional to the displacement of the body from its mean position. The system that performs simple harmonic motion is called the harmonic oscillator.

Case 1: The potential energy is zero, and the kinetic energy is maximum at the equilibrium point where zero displacement takes place.

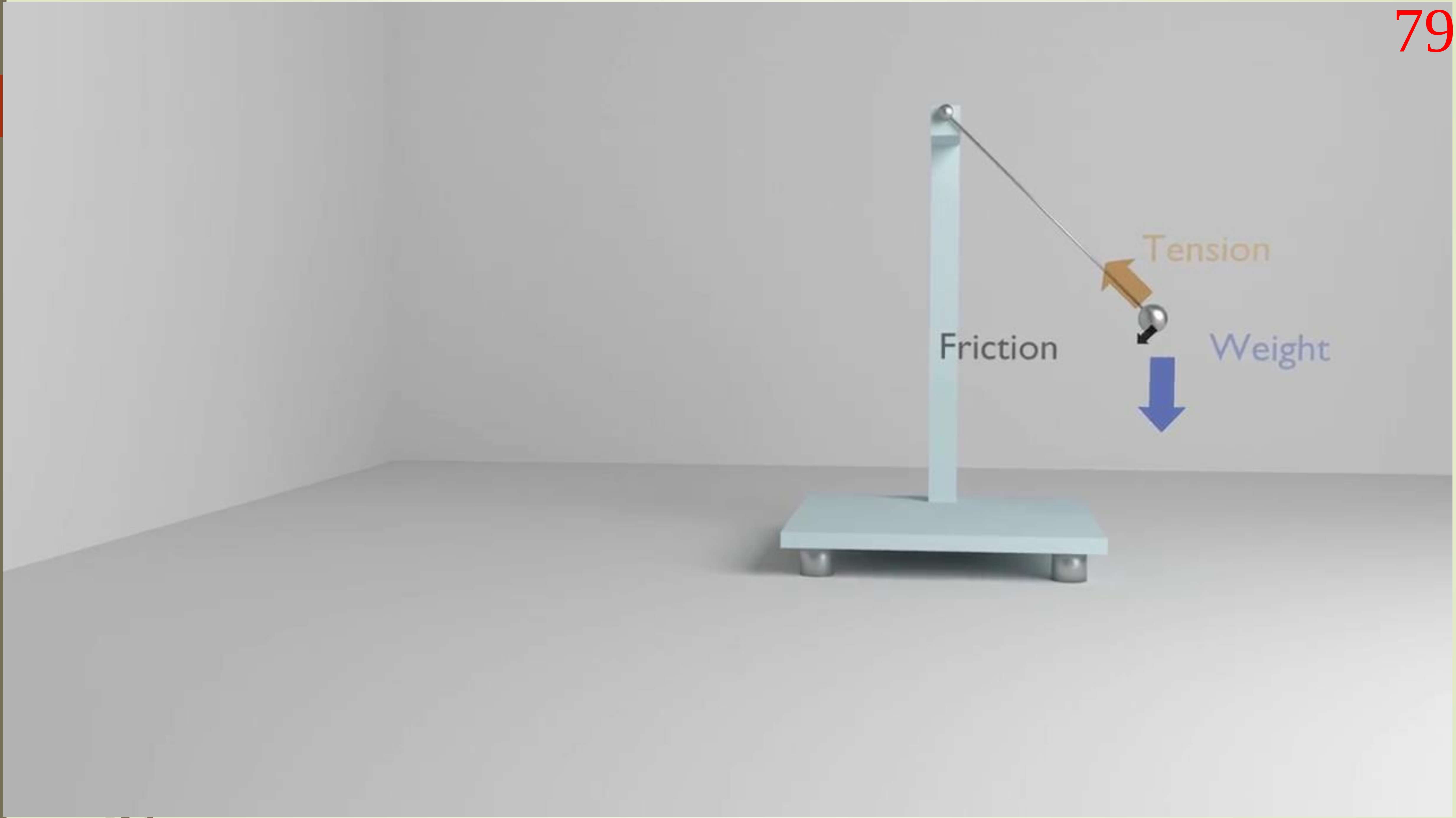
Case 2: The potential energy is maximum, and the kinetic energy is zero, at a maximum displacement point from the equilibrium point.

Case 3: The motion of the oscillating body has different values of potential and kinetic energy at other points.

Consider a particle of mass m, executing linear simple harmonic motion with angular frequency(ω) and amplitude(A).

$$x = A\,sin(\omega t + \varphi) \text{ and}$$

$$v = \frac{dx}{dt} = \frac{d}{dt}(A\sin(\omega t+\phi)) = A\omega\cos(\omega t+\phi).$$


Tension
Friction
Weight

Kinetic Energy of a Particle in Simple Harmonic Motion

$$\text{Kinetic Energy} = \frac{1}{2}mv^2 \quad \left[Since,\ v^2 = A^2\omega^2\cos^2(\omega t + \phi)\right]$$
$$= \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)$$

$$\therefore \text{Kinetic Energy} = \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)$$

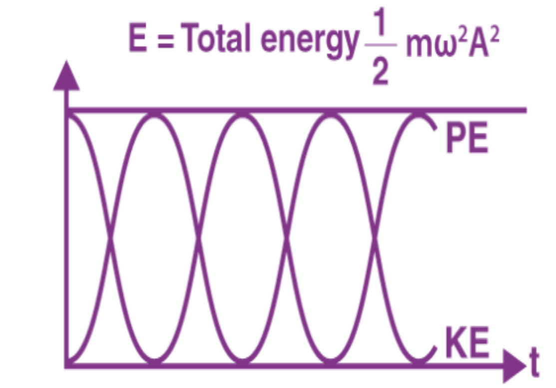


fig. (PE-KE-t) graph

Potential Energy of Simple Harmonic Motion

Work done by the restoring force while displacing the particle from the mean position (x = 0) to x = x:

The work done by restoring force when the particle has been displaced from the position x to x + dx is given by

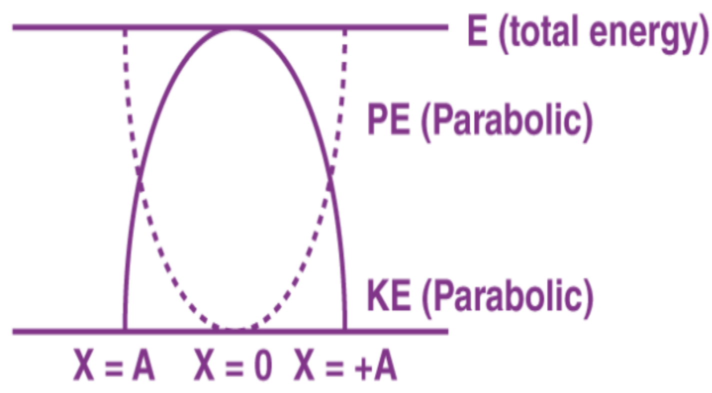
dw = F dx = -kx dx

$$w = \int dw = \int_0^x -kx\,dx = \frac{-kx^2}{2}$$
$$= -\frac{m\omega^2x^2}{2}$$
$$[k = m\omega^2]$$
$$= -\frac{m\omega^2}{2}A^2\sin^2(\omega t + \phi)$$

Potential Energy = -(work done by restoring force)

$$\text{Potential Energy} = \frac{m\omega^2x^2}{2} = \frac{m\omega^2A^2}{2}\sin^2(\omega t + \phi)$$

Variation of kinetic energy and potential energy in SHM with displacement:



The total energy of the system of a block and a spring is equal to the sum of the potential energy stored in the spring plus the kinetic energy of the block.

$$E_{Total}=K+U$$

$$E_{\text{Total}} = \frac{1}{2}kA^2\cos^2(\omega t+\varphi)+\frac{1}{2}mA^2\omega^2\sin^2(\omega t+\varphi)$$

$$= \frac{1}{2}kA^2\cos^2(\omega t+\varphi)+\frac{1}{2}kA^2\sin^2(\omega t+\varphi)$$
$$= \frac{1}{2}kA^2\left(\cos^2(\omega t+\varphi)+\sin^2(\omega t+\varphi)\right)$$
$$= \frac{1}{2}kA^2.$$

The total energy of the system of a block and a spring is equal to the sum of the potential energy stored in the spring plus the kinetic energy of the block and is proportional to the square of the amplitude $E_{\text{Total}} = (1/2)\,kA^2$. The total energy of the system is constant.

Problem 01

A 200-g block connected to a light spring for which the force constant is 5.00 N/m is free to oscillate on a frictionless, horizontal surface. The block is displaced 5.00 cm from equilibrium and released from rest as in Figure 15.6.

(A) Find the period of its motion.

Solution:

the angular frequency of the

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00\ \text{N/m}}{200 \times 10^{-3}\ \text{kg}}} = 5.00\ \text{rad/s}$$

1 the period of the system:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00\ \text{rad/s}} = 1.26\ \text{s}$$

Problem 02

A 0.500-kg cart connected to a light spring for which the force constant is 20.0 N/m oscillates on a frictionless, horizontal air track.

(A) Calculate the maximum speed of the cart if the amplitude of the motion is 3.00 cm.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\max}^2$$

$$v_{\max} = \sqrt{\frac{k}{m}}\,A = \sqrt{\frac{20.0\ \text{N/m}}{0.500\ \text{kg}}}\,(0.030\,0\ \text{m}) = 0.190\ \text{m/s}$$

The simple Pendulum

It consists of a particle-like bob of mass *m* suspended by a light string of length *L* that is fixed at the upper end as shown in Fig. Newton's second law for motion:

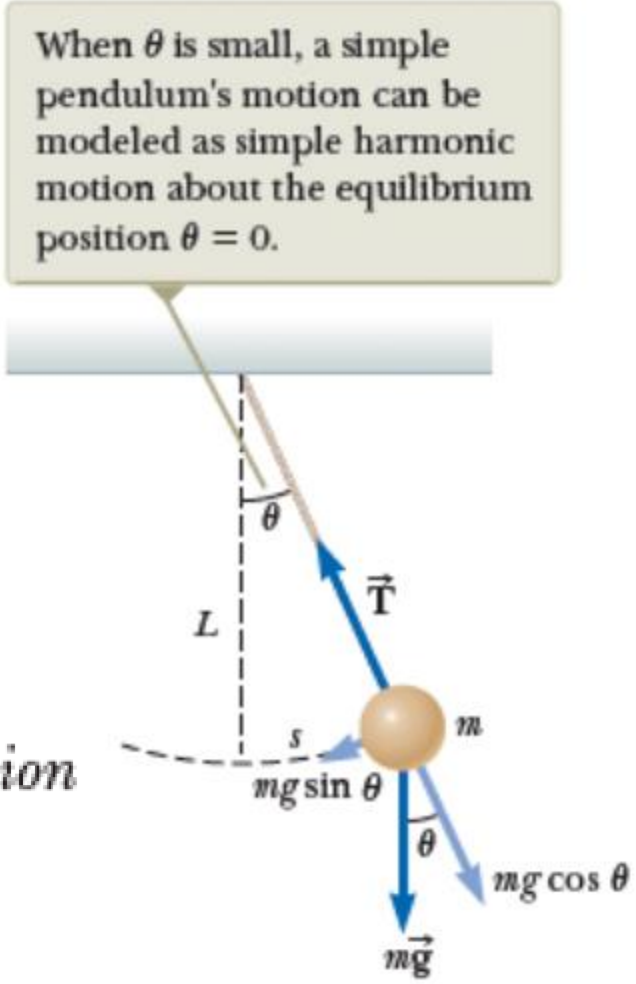
$$F_t = ma_t \rightarrow -mg\sin\theta = m\frac{d^2s}{dt^2}$$ Because $s = L\theta$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$ Or, $$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$ (for small values of θ)

$\theta = \theta_{max}\cos(\omega t + \phi)$, θ_{max} is the *maximum angular position*

and the angular frequency ω is $\omega = \sqrt{\frac{g}{L}}$

The period of the motion is $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$



Problem 03

Christian Huygens (1629–1695), the greatest clockmaker in history, suggested that an international unit of length could be defined as the length of a simple pendulum having a period of exactly 1 s. How much shorter would our length unit be if his suggestion had been followed?

$$L = \frac{T^2 g}{4\pi^2} = \frac{(1.00\ \text{s})^2(9.80\ \text{m/s}^2)}{4\pi^2} = \boxed{0.248\ \text{m}}$$

Problem 04

WHAT IF? What if Huygens had been born on another planet? What would the value for g have to be on that planet such that the meter based on Huygens's pendulum would have the same value as our meter?

$$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(1.00\ \text{m})}{(1.00\ \text{s})^2} = 4\pi^2\ \text{m/s}^2 = 39.5\ \text{m/s}^2$$

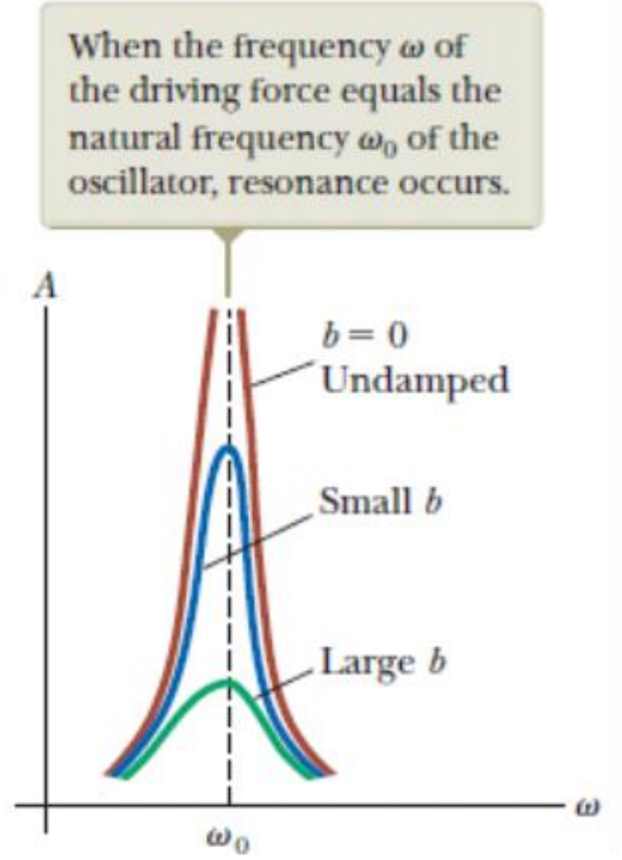
No planet in our solar system has an acceleration due to gravity that large.

Forced Oscillations

A common example of a forced oscillator is a damped oscillator driven by an external force that varies periodically, $F(t) = F_0 \sin \omega t$, where F_0 is a constant

The solution of this equation is rather lengthy and will not be presented. $x = A\cos(\omega t + \phi)$

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0{}^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$ where $\omega_0 = \sqrt{k/m}$



The dramatic increase in amplitude near the natural frequency is called **resonance,** and the natural

$_0$ is also called the **resonance frequency** of the system.

❖ Forced oscillations:

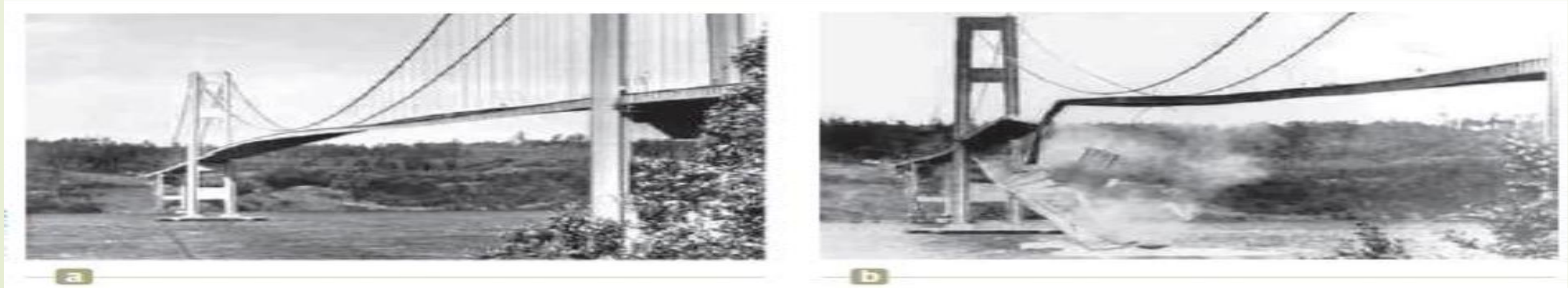
Forced oscillations occur when an oscillating system is driven by a periodic force that is external to the oscillating system.

❖ Undamped Oscillation:

When the frictional dissipation of energy is neglected, the motion is said to be ***undamped oscillation.***

❖ What is resonance:

When the frequency of an externally applied periodic force of a body is equal to the natural frequency of this body then the body readily begins to vibrate or free to vibrate with an increased amplitude. This phenomenon is known as resonance.



(a) In 1940, turbulent winds set up torsional vibrations in the Tacoma Narrows Bridge, causing it to oscillate at a frequency near one of the natural frequencies
bridge's collapse. (Mathematicians and physicists are currently challenging some aspects of this interpretation.)

Problem 05

A simple harmonic oscillator can be described by $x = 0.3 \sin(\omega t + \alpha)$ m. It has a mass of 30 kg and a force constant of 480 $N.m^{-1}$. Also, x = 0.2 m at time t = 0.1 s. >Find

(a) the maximum mechanical energy of the oscillator,

(b) the potential energy and kinetic energy when the displacement is 0.2 m,

(c) the angular frequency of the oscillation,

(d) the initial phase of the displacement.

Problem 05

A simple harmonic oscillator can be described by $x = 0.3\sin(\omega t + \alpha)$ m. It has a mass of 30 kg and a force constant of 480 $N.m^{-1}$. Also, x = 0.2 m at time t = 0.1 s. >Find

(a) the maximum mechanical energy of the oscillator,

(b) the potential energy and kinetic energy when the displacement is 0.2 m,

(c) the angular frequency of the oscillation,

(d) the initial phase of the displacement.

(a)

$$\left.\begin{matrix} maximum \\ mechanical \\ energy \end{matrix}\right\} = \frac{1}{2}kA^2 = \frac{1}{2}\times 480\times 0.3^2 = 21.6J$$

(b)

$$\left.\begin{matrix} potential \\ energy \end{matrix}\right\} = \frac{1}{2}kx^2 = \frac{1}{2}\times 480\times 0.2^2 = 9.6J$$

and

$$\left.\begin{matrix} kinetic \\ energy \end{matrix}\right\} = \frac{1}{2}kA^2 - \frac{1}{2}kx^2 = 21.6 - 9.6 = 12.0J$$

(c)

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{480}{30}} = 4\ \text{rad.s}^{-1}$$

(d)

$$x = A\cdot\sin(\omega t + \alpha)$$

$$0.2 = 0.3\cdot\sin(4\times 0.1 + \alpha)$$

$$\sin(0.4 + \alpha) = \frac{0.2}{0.3} = 0.666$$

$$0.4 + \alpha = 0.73\ \text{rad}$$

$$\alpha = 0.33\ \text{rad}$$

Problem 06: A particle oscillating along a straight line in a simple harmonic motion has amplitude 0.05 m .The time period of oscillation is 12 second. calculate maximum speed and acceleration

Problem 07: The maximum velocity of a particle executing simple harmonic motion is 6.24 cm per second. if the amplitude of the particle is 3 cm what is its time period

Problem 08: An object is in simple harmonic motion has amplitude 0.01 m and a frequency 12 Hz. calculate its velocity at displacement of 0.005 m.

Problem 09: Body of mass 50 gm is attached with on end of the spring and his allowed to oscillate in SHM. The amplitude of the motion is 12 cm and time period is 1.70s.Calculate the (i) frequency (ii) Spring constant (iii) maximum velocity of the body (iv)Its maximum acceleration (v) speed when displacement is 6 cm and (vi)acceleration when x is equal to 6 cm.

Problem 10: A simple pendulum of effective length on 1 meter completes 2 oscillations per second. what is the magnitude of acceleration due to gravity

Problem 11: Find the length of a second pendulum at the acceleration due to gravity is 9.8 ms^{-2}

Problem 12: The maximum velocity of a particle executing simple harmonic motion is 0.2 cm per second if its amplitude is 0.004 cm. then what is its time period

Problem 13: Length of the thread of simple pendulum is 98 cm and its time period 2 second calculate the radius of the bob

3rd Week

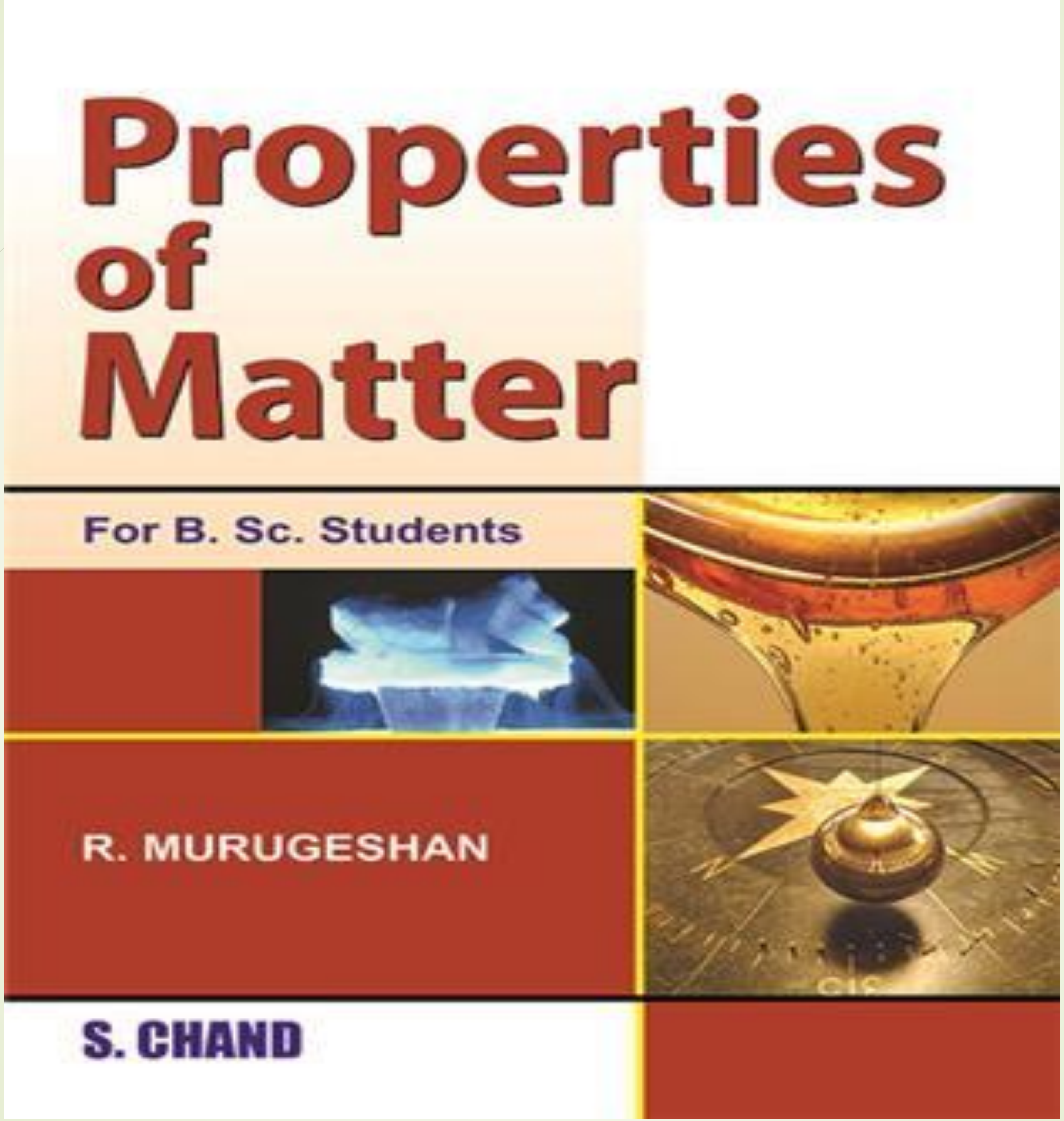
Topic:

Properties of Matter:

Elasticity,

Topic Related Math

Page: 90- 107


Properties of Matter
For B. Sc. Students
R. MURUGESHAN
S. CHAND

What is Stress?

Stress is defined as force per unit area within materials that arise from externally applied forces, uneven heating, or permanent deformation and that permits an accurate description and prediction of elastic, plastic, and fluid behavior.

Stress is given by the following formula: $\sigma = \frac{F}{A}$

where σ is the stress applied, F is the force applied, and A is the area of the force application.

The unit of stress is N/m^2.

❖ What is Strain?

Strain is the amount of deformation experienced by the body in the direction of force applied, divided by the initial dimensions of the body.

The following equation gives the relation for deformation in terms of the length of a solid:

$$\epsilon = \frac{\delta l}{L}$$

where ε is the strain due to the stress applied, δl is the change in length and L is the original length of the material.

The strain is a dimensionless quantity as it just defines the relative change in shape.

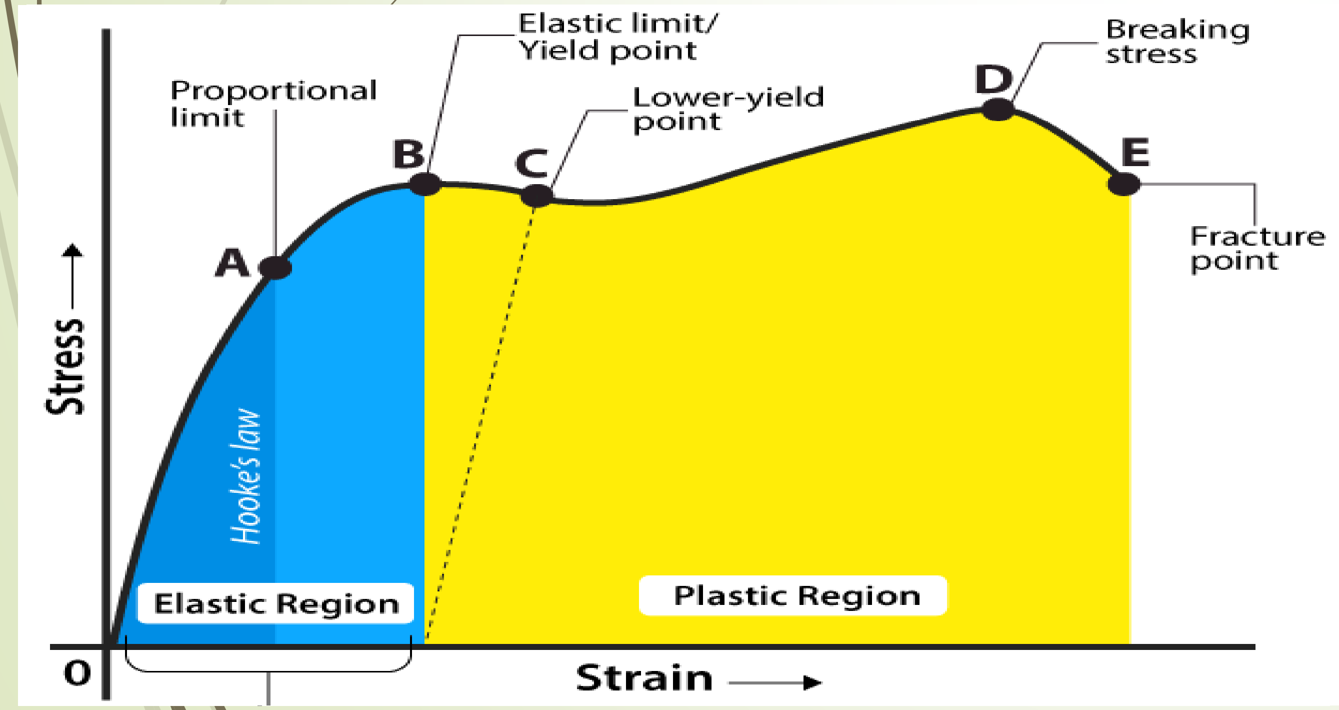
❖ Elasticity

In physics and materials science, elasticity is the ability of a body to resist a distorting influence and to return to its original size and shape when that influence or force is removed.

❖ Explaining Stress-Strain Graph

We can learn about the elastic properties of materials by studying the stress-strain relationships, under different loads, in these materials.

In a stress-strain curve, the stress and its corresponding strain values are plotted. An example of a stress-strain curve is given below.



The different regions in the stress-strain diagram are:

(i) Proportional Limit

It is the region in the stress-strain curve that obeys Hooke's Law. In this limit, the stress-strain ratio gives us a proportionality constant known as Young's modulus. The point OA in the graph represents the proportional limit.

(ii) Elastic Limit

It is the point in the graph up to which the material returns to its original position when the load acting on it is completely removed. Beyond this limit, the material doesn't return to its original position, and a plastic deformation starts to appear in it.

(iii) Yield Point

The yield point is defined as the point at which the material starts to deform plastically. After the yield point is passed, permanent plastic deformation occurs. There are two yield points (i) upper yield point (ii) lower yield point.

(iv) Ultimate Stress Point

It is a point that represents the maximum stress that a material can endure before failure. Beyond this point, failure occurs.

(v) Fracture or Breaking Point

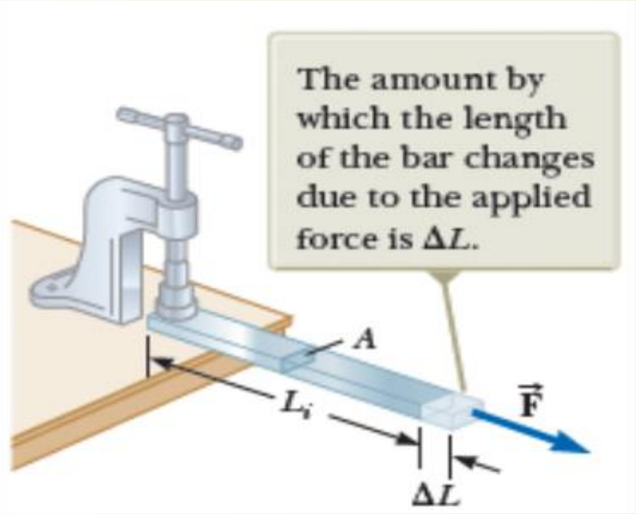
It is the point in the stress-strain curve at which the failure of the material takes place.

Hooke's law:

Within the elastic limit, stress is directly proportional to strain.

$$\begin{aligned} Stress &\propto Strain \\ Stress &= E \times Strain \\ E &= \frac{Stress}{Strain} \end{aligned}$$

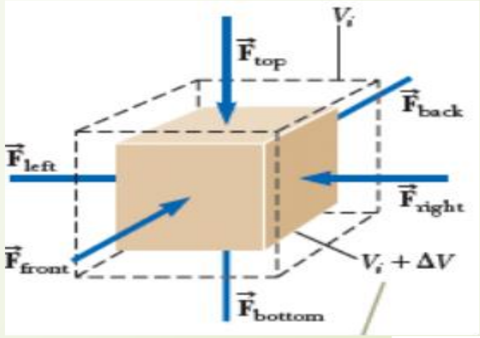
E is a constant called the modulus of elasticity. There are three types of modulus:



Young's modulus:

When the force is applied to the body only along a particular direction, the change per unit length in that direction is called longitudinal. Within the elastic limit, the ratio of normal stress to longitudinal strain is called Young's modulus.

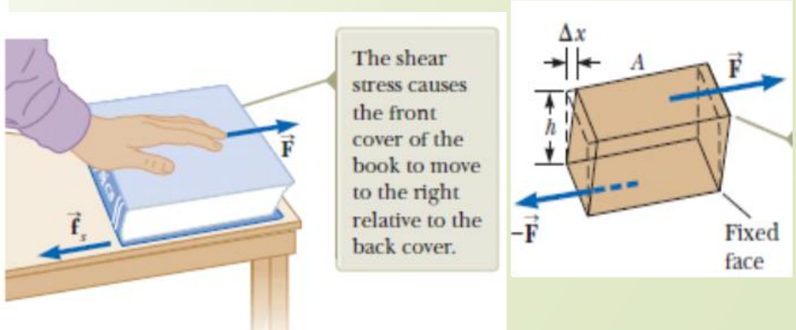
Young's modulus, $Y = \frac{Normal\,stress}{Lonitudinal\,strain} = \frac{\frac{F}{A}}{\frac{l}{L}} = \frac{FL}{Al}$



Bulk modulus:

When the force is applied normally and uniformly to the whole surface of the body, so that there is a change of volume. Within the elastic limit, the ratio of normal stress to volume strain is called Bulk modulus.

Bulk modulus, $K = \frac{Normal\,stress}{Volume\,strain} = \frac{\frac{F}{A}}{\frac{v}{V}} = \frac{FV}{Av} = \frac{PV}{v}$



Modulus of rigidity:

When the tangential force is applied to the body, a change in the inclinations of the coordinate axes of the system or the body occur in geometrically. Within the elastic limit, the ratio of tangential stress to shearing strain is called Modulus of rigidity.

Modulus of rigidity, $\eta = \frac{Tangential\,stress}{Shearing\,strain} = \frac{\frac{F}{A}}{\theta} = \frac{F}{A\theta}$

Work done for longitudinal strain:

Consider a wire of length L, area of cross-section A and Young's modulus of elasticity Y. Let l be the increase in length when a force F is applied.
Therefore, Work done

$$\int dW = \int_0^l F dl$$

We know, $Y = \frac{FL}{Al}$
So, $F = \frac{YAl}{L}$
Now,

$$\begin{aligned} W &= \frac{YA}{L}\int_0^l l dl \\ &= \frac{YAl^2}{2L} \\ &= \frac{1}{2}(\frac{YAl}{L})l \\ &= \frac{1}{2}Fl \end{aligned}$$

Work done per unit volume,

$$\begin{aligned} w &= \frac{W}{V} = \frac{W}{AL} \\ &= \frac{Fl}{2AL} = \frac{1}{2}\frac{F}{A}\frac{l}{L} \\ &= \frac{1}{2} \times Stress \times Strain \end{aligned}$$

Work done for volume strain:

Consider a cube of volume V, area of cross-section A and length L. When a normal stress P is applied, the change in volume is v.

Therefore, Work done

$$\int dW = \int_0^v P dv$$

We know, $K = \frac{FV}{Av}$ and $P = \frac{F}{A}$

So, $P = \frac{Kv}{V}$

Now,

$$\begin{aligned} W &= \frac{K}{V} \int_0^v v dv \\ &= \frac{Kv^2}{2V} \\ &= \frac{1}{2}(\frac{Kv}{V})v \\ &= \frac{1}{2}Pv \end{aligned}$$

Work done per unit volume,

$$\begin{aligned} w &= \frac{W}{V} \\ &= \frac{Pv}{2V} = \frac{1}{2}\frac{F}{A}\frac{v}{V} \\ &= \frac{1}{2} \times Stress \times Strain \end{aligned}$$

Work done for shearing strain:

Consider a cube of side L. When a tangential force F is applied to the upper face of the cube, the cube is sheared through an angle θ. If the tangential stress is T, then

$$\begin{aligned} \eta &= \frac{T}{\theta} \\ T &= \eta\theta \\ \frac{F}{A} &= \eta\theta \\ F &= \eta\theta A \end{aligned}$$

Total tangential force, $F = \eta\theta A$
Therefore, Work done

$$\int dW = \int_0^{\theta} F dl$$

Shearing strain, $d\theta = \frac{dl}{L}$ So, $dl = Ld\theta$
Now,

$$
\begin{aligned}
W &= \int_0^\theta \eta\theta AL d\theta \\
&= \eta AL \int_0^\theta \theta d\theta \\
&= \frac{\eta AL\theta^2}{2}
\end{aligned}
$$

Work done per unit volume,

$$
\begin{aligned}
w &= \frac{W}{V} = \frac{W}{AL} \\
&= \frac{\eta AL\theta^2}{2AL} = \frac{1}{2}\eta\theta^2 \\
&= \frac{1}{2}\eta\theta \times \theta \\
&= \frac{1}{2}T\theta \\
&= \frac{1}{2} \times Stress \times Strain
\end{aligned}
$$

Lateral strain:

Whenever a body is subjected to a force in a particular direction, there is change in dimensions of the body in the other two perpendicular directions, This is called lateral strain.

Poisson's ratio:

Let α be the longitudinal strain per unit stress and β the lateral strain per unit stress. Within the elastic limit,

$$\begin{aligned} \beta &\propto \alpha \\ \beta &= \sigma\alpha \\ \therefore \sigma &= \frac{\beta}{\alpha} \end{aligned}$$

σ is called Poisson's ratio. So poisson's ratio is the ratio of lateral strain per unit stress to the longitudinal strain per unit stress.

Maximum value of poisson's ratio:

Consider a wire of length L and diameter D. The wire is fixed at one end and a force is applied at the other end. Consequently the length of the wire increases and the diameter of the wire decreases.

Increase in length $= dL$
Decrease in diameter $= -dD$
So,

$$\begin{aligned} \sigma &= \frac{\beta}{\alpha} \\ &= -\frac{\frac{dD}{D}}{\frac{dL}{L}} \\ &= -\left(\frac{dD}{dL}\right)\left(\frac{L}{D}\right) \end{aligned}$$

If the volume of the wire remains unchanged after the force has been applied.
Initial volume of the wire, $V = \frac{\pi D^2 L}{4}$
Differentiating this equation, we can write

$$\begin{aligned} dV &= \frac{\pi}{4}\Big[D^2 dL + 2LDdD\Big] \\ D^2 dL + 2LDdD &= 0 \\ D^2 dL &= -2LDdD \\ -\left(\frac{dD}{dL}\right) \times \left(\frac{L}{D}\right) &= \frac{1}{2} \\ \sigma &= \frac{1}{2} \end{aligned}$$

This is the maximum possible value of poisson's ratio.

1. Explain the relationship between stress and strain in the context of elastic modulus.

How would you calculate the elastic modulus for a material given a stress-strain graph?

2. Why do you think understanding the elastic modulus of materials is essential for designing safe structures like bridges or skyscrapers? Share an example where this property might have a significant impact.

3. Using a tensile testing machine, demonstrate how to measure the elastic modulus of a metal rod. Record the stress and strain values and plot a graph to determine the modulus.

❖ **Cognitive Domain (Understanding and Applying)**

Question:
Explain the relationship between stress and strain in the context of elastic modulus. How would you calculate the elastic modulus for a material given a stress-strain graph?

Purpose:
This question evaluates students' understanding of the concept and their ability to apply knowledge to interpret data.

Affective Domain (Valuing and Responding)

Question:

Why do you think understanding the elastic modulus of materials is essential for designing safe structures like bridges or skyscrapers? Share an example where this property might have a significant impact.

Purpose:

This question assesses students' ability to recognize the importance of the concept in real-world applications and reflect on its significance.

Psychomotor Domain (Practical Application)

Question:

Using a tensile testing machine, demonstrate how to measure the elastic modulus of a metal rod. Record the stress and strain values and plot a graph to determine the modulus.

Purpose:

This hands-on task evaluates students' ability to perform an experiment, collect data, and interpret the results accurately.

Table 12.1 Typical Values for Elastic Moduli

Substance	Young's Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Problem 14

Now suppose the tension in the cable is 940 N as the actor reaches the lowest point. What diameter should a 10-m-long steel cable have if we do not want it to stretch more than 0.50 cm under these conditions? (y=20×10^{10} N/m^2)

Solution : Assuming cross-section is circular, find the diameter of the cable from $d = 2r$ and $A = \pi r^2$:

$$A = \frac{FL_i}{Y\Delta L}$$

$$d = 2r = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{FL_i}{\pi Y \Delta L}} = 2\sqrt{\frac{(940\ \text{N})(10\ \text{m})}{\pi(20 \times 10^{10}\ \text{N/m}^2)(0.005\,0\ \text{m})}}$$

Problem 15.

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is 1.0× 10^5 N/m^2(normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is 2.0 × 10^7 N/m^2. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged ?. Here Bulk Modulus = 6.1 × 10^{10} N/m^2

Solution:

$$\Delta V = -\frac{V_i \Delta P}{B} = -\frac{(0.50\ \text{m}^3)(2.0 \times 10^7\ \text{N/m}^2 - 1.0 \times 10^5\ \text{N/m}^2)}{6.1 \times 10^{10}\ \text{N/m}^2} = -1.6 \times 10^{-4}\ \text{m}^3$$

Now suppose the tension in the cable is 940 N as the actor reaches the lowest point. What diameter should a 10-m-long steel cable have if we do not want it to stretch more than 0.50 cm under these conditions? (y=20 $\times 10^{10}$ N/m^2)

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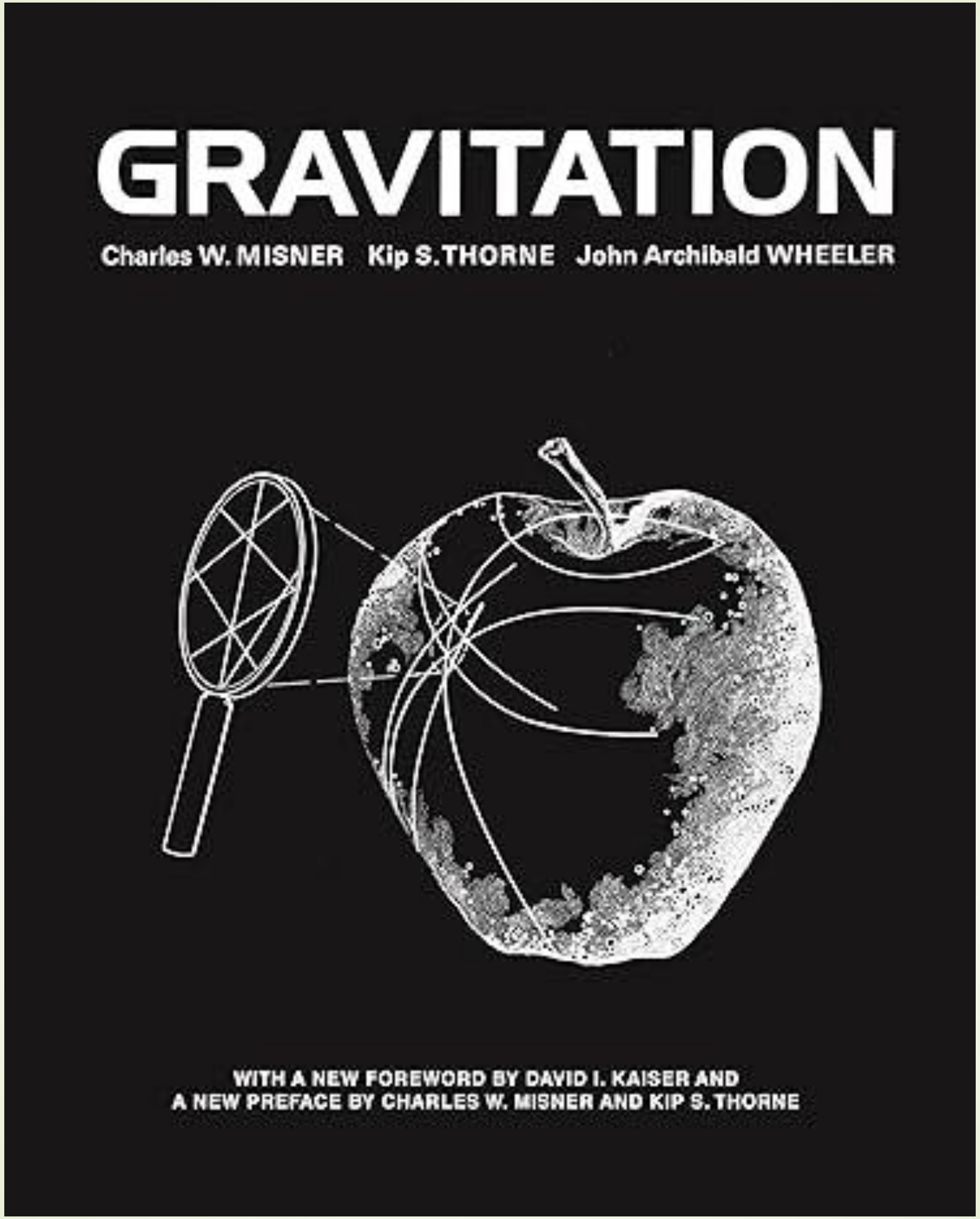
4TH WEEK

TOPIC: PROPERTIES OF MATTER,

GRAVITY RELATED PHENOMENON

TOPIC RELATED MATH

PAGE: 108- 122


GRAVITATION
Charles W. MISNER Kip S. THORNE John Archibald WHEELER
WITH A NEW FOREWORD BY DAVID I. KAISER AND
A NEW PREFACE BY CHARLES W. MISNER AND KIP S. THORNE

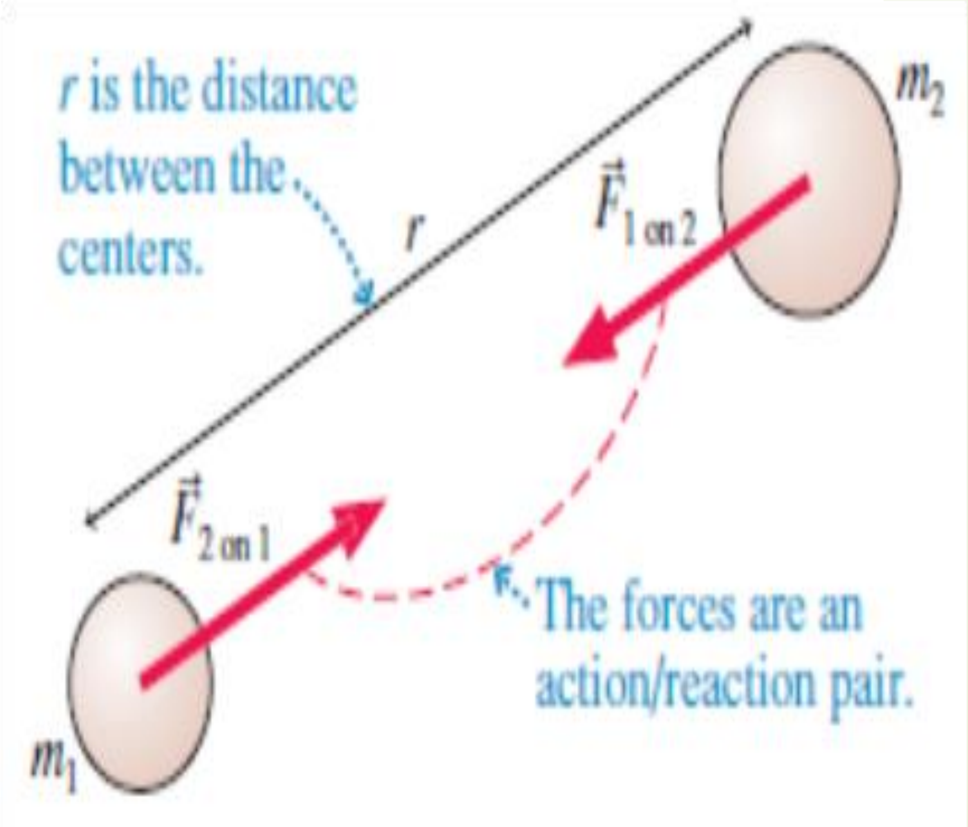
❖ Newton's Law of Universal Gravitation

Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. If the particles have masses $m1$ and $m2$ and are separated by a distance r, the magnitude of this gravitational force is

$$F_g = G\frac{m_1 m_2}{R^2}$$

where G is a constant, called the universal gravitational constant. Its value in SI unit

$$G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$



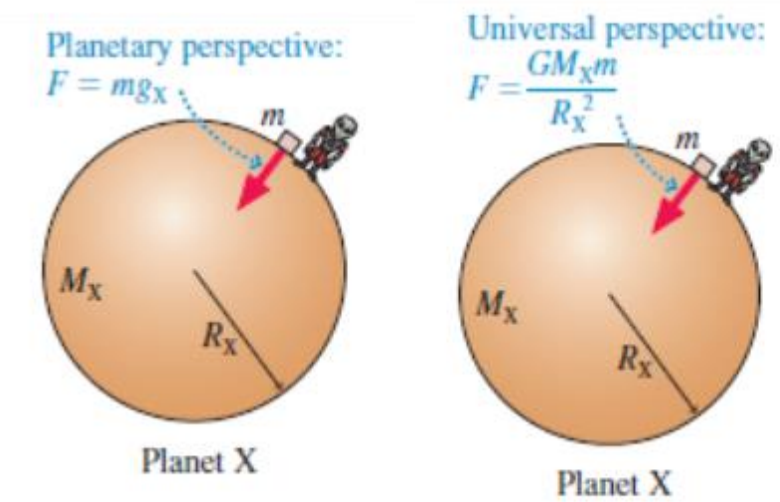
Free-Fall Acceleration and the Gravitational Force

$$mg = G\frac{M_E m}{R_E^2}$$

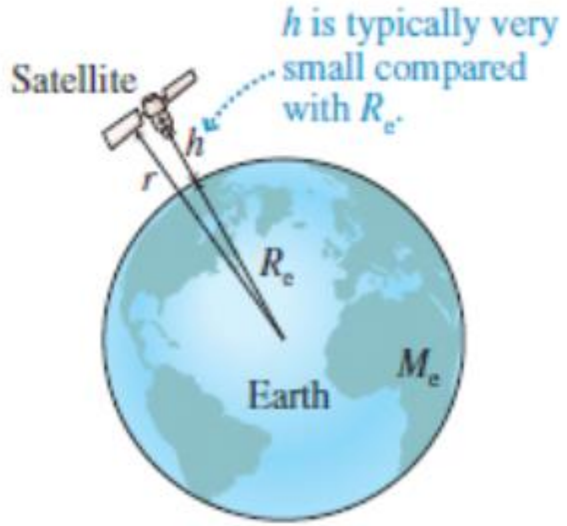
$$g = G\frac{M_E}{R_E^2}$$

$$F_g = mg$$

$$F_g = G\frac{M_E m}{R_E^2}$$



Now consider an object of mass *m* located a distance *h* above the Earth's surface or a distance *r* from the Earth's center, where $r = R_E + h$. The magnitude of



$$F_g = G\frac{M_E m}{r^2} = G\frac{M_E m}{(R_E + h)^2}$$

Problem 16.

Using the known radius of the Earth and that g =$\mathbf{9.80}$ $\boldsymbol{m/s^2}$ at the Earth's surface, find the average density of the Earth.

$$M_E = \frac{gR_E^2}{G}$$

$$\rho_E = \frac{M_E}{V_E} = \frac{gR_E^2/G}{\frac{4}{3}\pi R_E^3} = \frac{3}{4}\frac{g}{\pi G R_E}$$

$$= \frac{3}{4}\frac{9.80\ \text{m/s}^2}{\pi(6.674\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.37\times10^6\ \text{m})}$$

$$= 5.50\times10^3\ \text{kg/m}^3$$

Free-Fall Acceleration *g* at Various Altitudes Above the Earth's Surface

Altitude h (km)	g (m/s^2)
1 000	7.33
2 000	5.68
3 000	4.53
4 000	3.70
5 000	3.08
6 000	2.60
7 000	2.23
8 000	1.93
9 000	1.69
10 000	1.49
50 000	0.13
∞	0

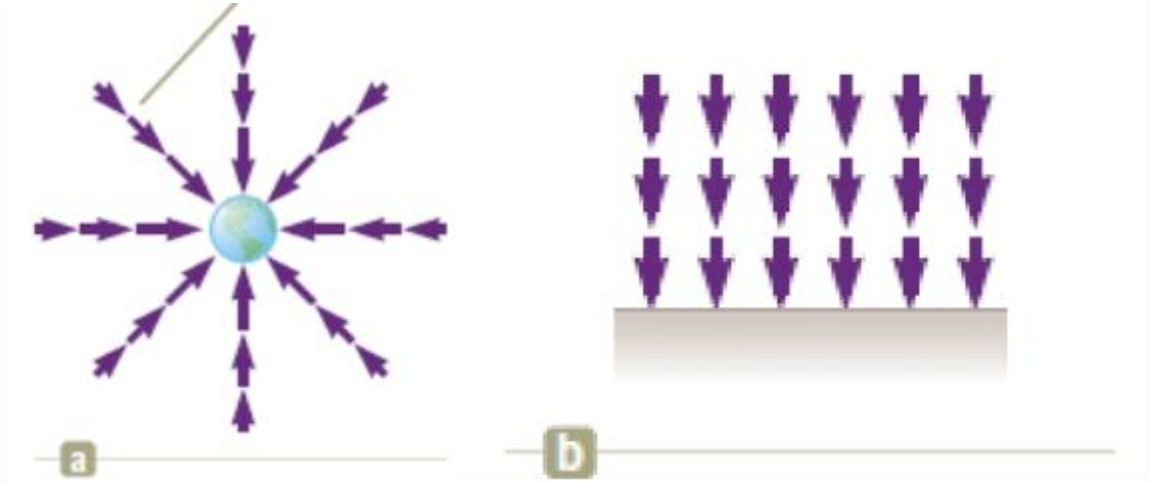
Analysis Model: Particle in a Field (Gravitational)

Gravitational field ▶

$$\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}_g}{m} = -\frac{GM_E}{r^2}\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector pointing radially outward from the Earth

that the field points toward the center of the Earth as illustrated in Figure a.



(a) The gravitational field vectors in the vicinity of a uniform spherical mass such as

magnitude. (b) The gravitational field vectors in a small region near the Earth’s surface are uniform in both direction and

Problem 17.

The International Space Station operates at an altitude of 350 km. Plans for the final construction show that material of weight 4.22×10^6 N, measured at the Earth's surface, will have been lifted off the surface by various spacecraft during the construction process. What is the weight of the space station when in orbit?

$$m = \frac{F_g}{g} = \frac{4.22 \times 10^6 \text{ N}}{9.80 \text{ m/s}^2} = 4.31 \times 10^5 \text{ kg}$$

$$g = \frac{GM_E}{(R_E + h)^2}$$

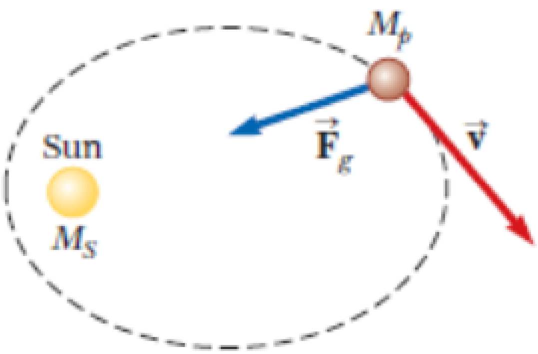
$$= \frac{(6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})}{(6.37 \times 10^6 \text{ m} + 0.350 \times 10^6 \text{ m})^2} = 8.82 \text{ m/s}^2$$

$$F_g = mg = (4.31 \times 10^5 \text{ kg})(8.82 \text{ m/s}^2) = 3.80 \times 10^6 \text{ N}$$

❖ Kepler's Laws and the Motion of Planets

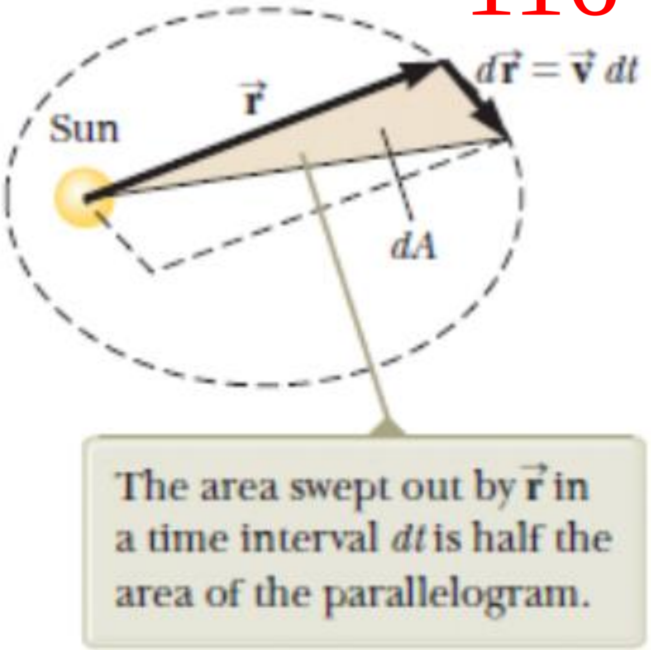
❑ Kepler's First Law:

All planets move in elliptical orbits with the Sun at one focus.



❑ Kepler's Second Law:

The radius vector is drawn from the Sun to a planet sweeps out equal areas in equal time intervals.



Evaluating $\vec{L}$ for the planet.

$$\vec{L} = \vec{r} \times \vec{p} = M_p \vec{r} \times \vec{v} \rightarrow L = M_p |\vec{r} \times \vec{v}|$$

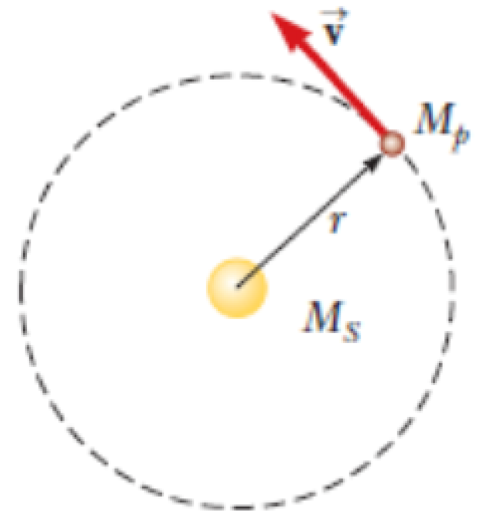
$$dA = \tfrac{1}{2}|\vec{r} \times d\vec{r}| = \tfrac{1}{2}|\vec{r} \times \vec{v}\,dt| = \tfrac{1}{2}|\vec{r} \times \vec{v}|\,dt$$

$$dA = \tfrac{1}{2}\left(\frac{L}{M_p}\right)dt$$

$$\frac{dA}{dt} = \frac{L}{2M_p}$$ where L and M_p are both constants.

❑ Kepler's Third Law:

The square of the orbital period of any planet is proportional to the cube of the semimajor axis of the elliptical orbit.



$$F_g = M_p a \quad \rightarrow \quad \frac{GM_S M_p}{r^2} = M_p\left(\frac{v^2}{r}\right)$$

$$\frac{GM_S}{r^2} = \frac{(2\pi r/T)^2}{r}$$

$$T^2 = \left(\frac{4\pi^2}{GM_S}\right) r^3 = K_S r^3$$

where K_S is a constant given by

$$K_S = \frac{4\pi^2}{GM_S} = 2.97 \times 10^{-19}\ \mathrm{s^2/m^3}$$

Problem 18.

Calculate the mass of the Sun, noting that the period of the Earth's orbit around the Sun is 3.156×10^7 s and its distance from the Sun is 1.496×10^{11} m.

$$M_S = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(1.496 \times 10^{11}\ \mathrm{m})^3}{(6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(3.156 \times 10^7\ \mathrm{s})^2} = \boxed{1.99 \times 10^{30}\ \mathrm{kg}}$$

5^{TH} WEEK

TOPIC: PROPERTIES OF MATTER:

GRAVITATION ENERGY RELATED PHENOMENON

TOPIC RELATED MATH

PAGE: 105-110

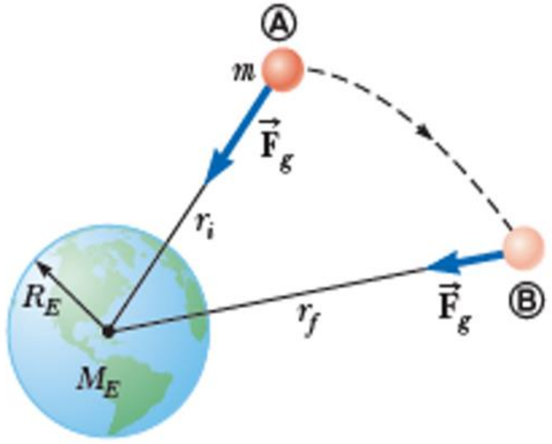
❖ Energy Considerations in Planetary and Satellite Motion

Satellites are launched from the earth to revolve around it. So, the energy required by a satellite to revolve around the earth is called its orbiting energy. Since this satellite revolves around the earth, it has kinetic energy and potential energy . The total energy of the satellite is calculated as the sum of the kinetic energy and the potential energy given by,

Total energy ,E = kinetic energy (K) + potential energy (U)

Potential Energy of Satellite:

As a particle of mass *m* moves from A to B above the Earth's surface, the gravitational potential energy of the particle–Earth system changes according to Equation:



$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)\, dr \qquad F(r) = -\frac{GM_E m}{r^2}$$

$$U_f - U_i = GM_E m \int_{r_i}^{r_f} \frac{dr}{r^2} = GM_E m \left[-\frac{1}{r}\right]_{r_i}^{r_f}$$

$$U_f - U_i = -GM_E m \left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

Taking $U_i = 0$ at $r_i = \infty$, we obtain the important result

$$U(r) = -\frac{GM_E m}{r}$$

Kinetic Energy of Satellite:

The two forces are acting on it are gravitational force, F_g and a centripetal force due to its velocity, F_c

Where $F_g = \frac{mM}{r^2}$ and $F_c = mv^2r$.

The magnitude of the forces is equal.

So, $F_g = F_c$

$$\frac{mM}{r^2} = mv^2r$$

$$\Rightarrow V^2 = \frac{GM}{r} \ldots (1)$$

We know that $K.E. = \frac{1}{2}mv^2$

Putting the value of V^2 in eq(1), we get,

K.E. of a satellite = $\frac{GMm}{2r}$

Total Energy of Satellite

The total energy of the satellite is calculated as the sum of the kinetic energy and the potential energy, given by,

T.E. = K.E. + P.E.

$$= \frac{GMm}{2r} - \frac{GMm}{r}$$

$$T.E. = -\frac{GMm}{2r}$$

❖ Escape Velocity :

In celestial mechanics, escape velocity or escape speed is the minimum speed needed for an object to escape from the earth's gravitational field.

❖ Derivation of Escape Velocity :

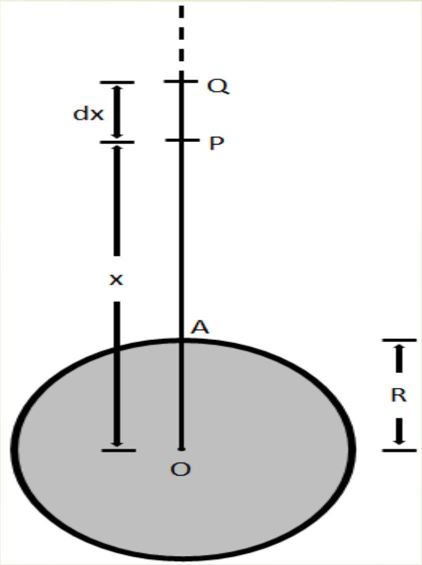
Suppose we have a sphere planet with radius **R** and mass **M**. And, a body of mass **m** is thrown from point **A** on the earth's surface. Let's join **OA** and draw it further. Consider two points, **P** and **Q**, with distance **x** and **(x + dx)** from the center of the Earth **O**.

Now, suppose the minimum velocity re… …ape from the Earth's surface is $\mathbf{v_e}$. Then, the kinetic energy of the object with mass **m** is

$$K.E. = \frac{1}{2} m v_e^2$$

Now, the work done to take the body f… …vitational force is given by

$$dW = F\,dx = \frac{GMm}{x^2}\,dx$$



we can calculate the total work done to take the body from the surface of the Earth to infinite against gravitational attraction by integrating the equation within limits: **x = R** to **x = ∞**.
Hence, the total work done is

$$W = \int_R^\infty dW = \int_R^\infty \frac{GMm}{x^2} dx$$

$$= GMm \int_R^\infty x^{-2} dx$$

$$= GMm \left[\frac{x^{-1}}{-1}\right]_R^\infty$$

$$= -GMm \left[\frac{1}{x}\right]_R^\infty = -GMm \left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$W = \frac{GMm}{R}$$

For the object to escape from the Earth's surface, the kinetic energy given must be equal to the work done against gravity going from the Earth's surface to infinity.

K.E. = W

Now, let's substitute the value of Kinetic Energy and work done against the earth's gravity.

K.E. = W

or, $\frac{1}{2} m v_e^2 = \frac{GMm}{R}$

$$v_e = \sqrt{\frac{2GM}{R}}$$

Since we know acceleration due to gravity is given by

$$g = \frac{GM}{R^2}$$

$$v_e = \sqrt{2gR}$$

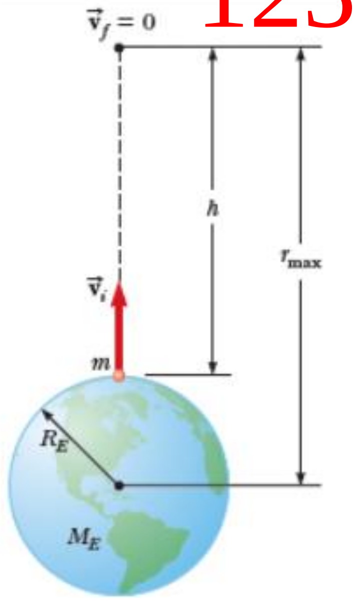
❖ Escape Speed

As the object is projected upward from the surface of the Earth,

$_i$ and $r = r_i = R_E$. When the object reaches its maximum

f f max .

$$\frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{max}} \quad \text{Letting } r_{max} \to \infty \quad v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$



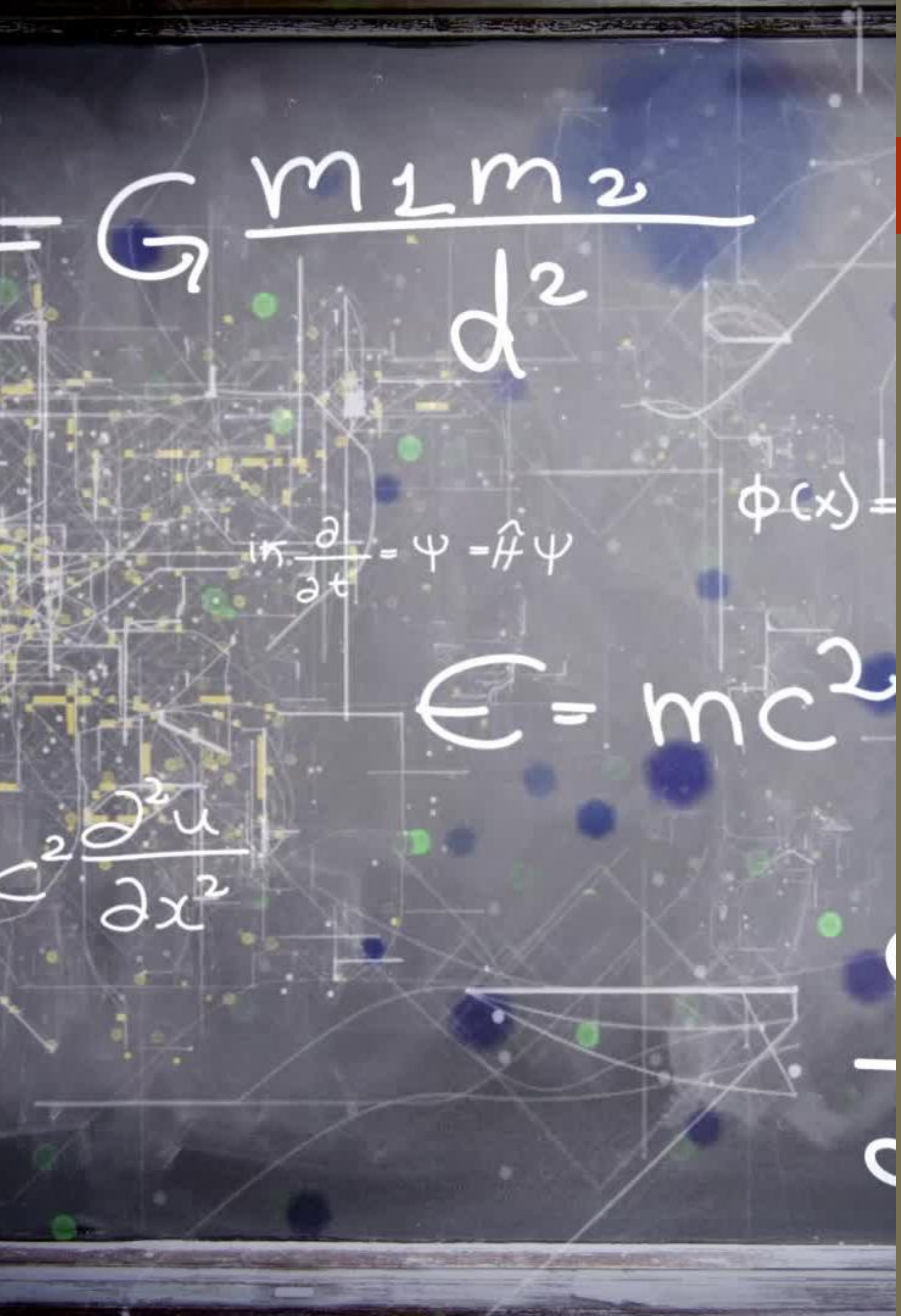
❑ Problem-19:

Calculate the escape speed from the Earth for a 5 000-kg object is spacecraft and determine the kinetic energy it must have at the Earth's surface to move infinitely far away from the Earth.

➢ Solution:

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2(6.674 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})}{6.37 \times 10^6\ \text{m}}} = 1.12 \times 10^4\ \text{m/s}$$

$$K = \frac{1}{2}mv_{esc}^2 = \frac{1}{2}(5.00 \times 10^3\ \text{kg})(1.12 \times 10^4\ \text{m/s})^2 = 3.13 \times 10^{11}\ \text{J}$$



6th Week

Topic: Properties of Matter:

Fluid Dynamics, Continuity Equation

Topic Related Math

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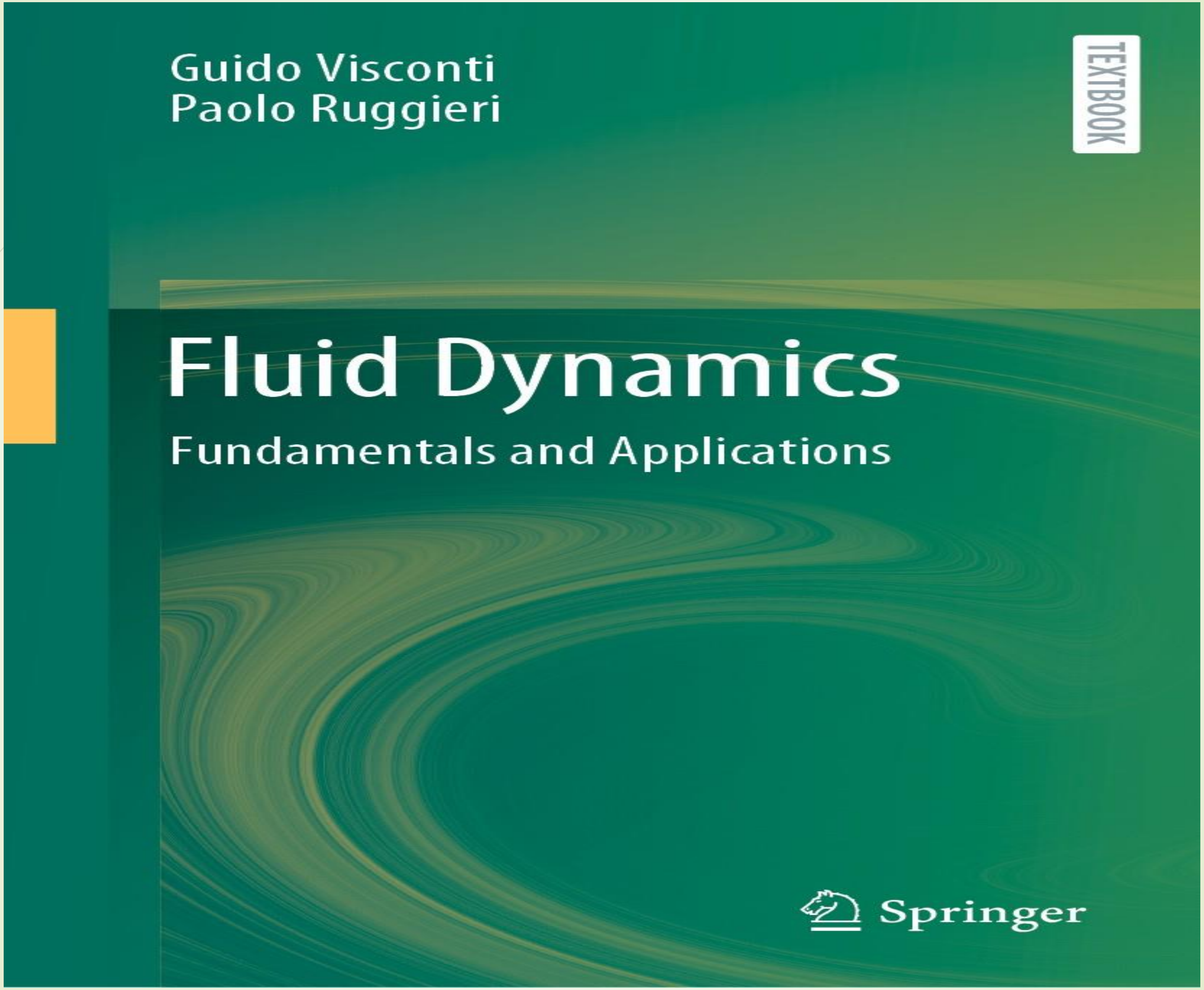

Guido Visconti
Paolo Ruggieri
TEXTBOOK
Fluid Dynamics
Fundamentals and Applications
Springer

❖ What is Fluid dynamics:

In physics, physical chemistry, and engineering, fluid dynamics is a subdiscipline of fluid mechanics that describes the flow of fluids—liquids and gases. It has several subdisciplines, including aerodynamics and hydrodynamics.

➢ Assumptions of ideal fluid flow :

The motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In our simplification model of ideal fluid flow, we make the following four assumptions:

1. The fluid is nonviscous.

   In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.

2. The flow is steady.

   In steady (laminar) flow, all particles passing through a point have the same velocity.

3. The fluid is incompressible.

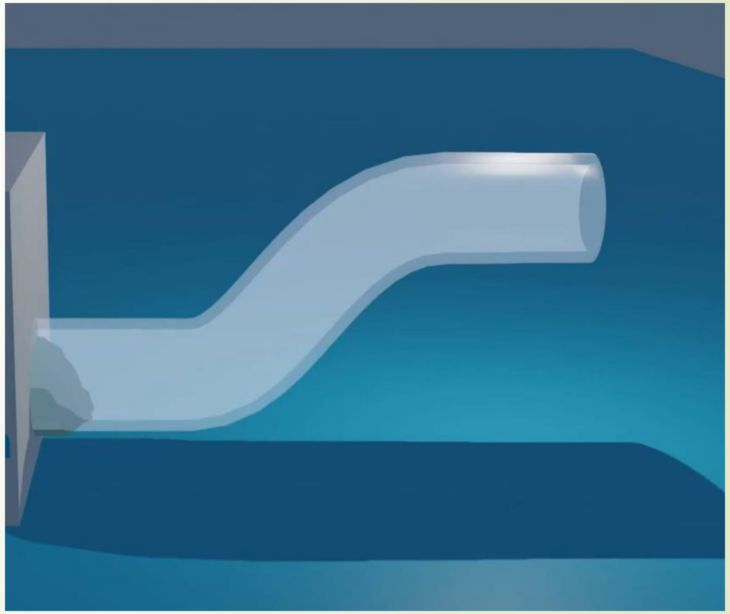
   The density of an incompressible fluid is constant.

4. The flow is irrotational.

   In irrotational flow, the fluid has no angular momentum at any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, the flow is irrotational.

❖ Describe The continuity equation

The continuity equation is a fundamental principle in physics that expresses the conservation of mass for a fluid or a current flowing in a given system. It states that the rate at which mass enters or leaves a specified volume of the system is equal to the rate of change of mass within that volume, taking into account any sources or sinks of mass within the volume.



The continuity equation can be mathematically expressed as follows:

$$\partial \rho / \partial t + \nabla \cdot (\rho v) = 0$$

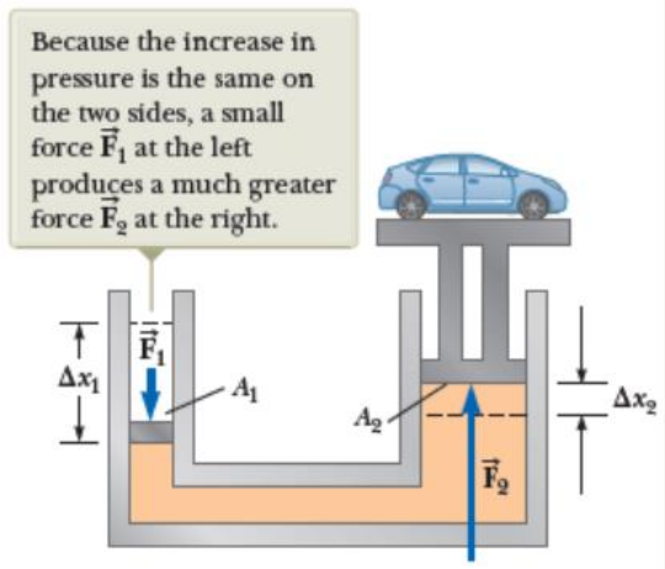
Where:

- $\partial \rho / \partial t$ is the rate of change of mass density (ρ) with respect to time (t).
- ∇ is the del operator, representing the gradient in three-dimensional space.
- (ρv) is the mass flux density, which is the product of mass density (ρ) and the velocity vector (v) of the fluid or current.

❖ Pascal's law:

a change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of th $F_1 \, \Delta x_1 = F_2 \, \Delta x_2.$

the work done by F_1 on the input piston equals the work done by F_2 on the output piston.



20-Example: In a car lift, compressed air exerts a force on a small piston that has a circular cross section of radius 5.00 cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0 cm. (A) What force must the compressed air exert to lift a car weighing 13 300 N? (B) What air pressure produces this force?

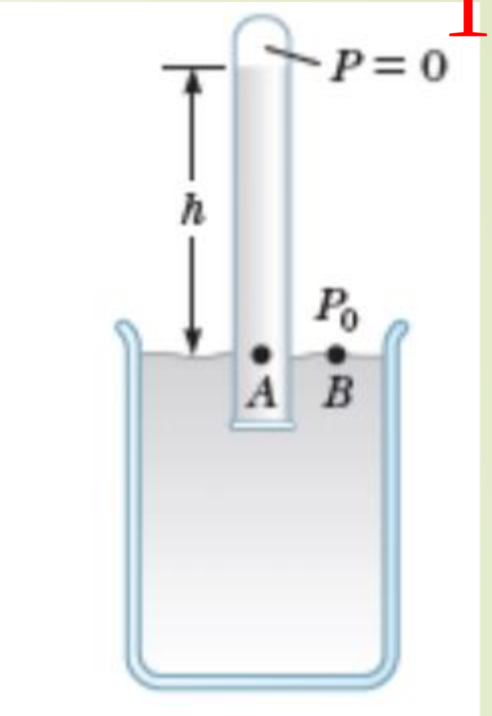
(A)

$$F_1 = \left(\frac{A_1}{A_2}\right)F_2 = \frac{\pi(5.00 \times 10^{-2}\text{ m})^2}{\pi(15.0 \times 10^{-2}\text{ m})^2}(1.33 \times 10^4\text{ N}) = 1.48 \times 10^3\text{ N}$$

(B)

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3\text{ N}}{\pi(5.00 \times 10^{-2}\text{ m})^2} = 1.88 \times 10^5\text{ Pa}$$

❖ Pressure Measurements: Torricelli experiment



A long tube closed at one end is filled with mercury and then inverted into a dish of mercury (Fig.). The closed end of the tube is nearly a vacuum, so the pressure at the top of the mercury column can be taken as zero. In Figure, the pressure at point *A*, due to the column of mercury, must equal the pressure at point *B*, due to the atmosphere. If that were not the case, there would be a net force that would move mercury from one point to the other until equilibrium is established. Therefore, $P = h\rho g$, where ρ Is the density of the mercury and *h* is the height of the mercury column.

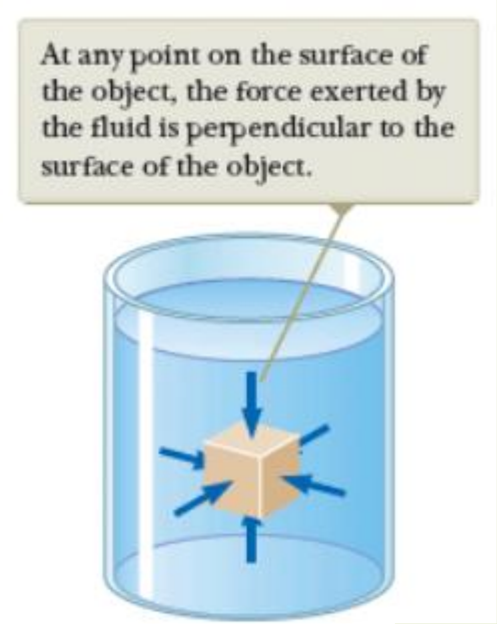
Let us determine the height of a mercury column for one atmosphere of pressure

$$P_0 = \rho_{Hg} g h \quad \rightarrow \quad h = \frac{P_0}{\rho_{Hg} g} = \frac{1.013 \times 10^5 \text{ Pa}}{(13.6 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 0.760 \text{ m}$$

❖ Buoyant Force and Archimedes' Principle:

The upward force exerted by a fluid on any immersed object is called a buoyant force. The magnitude of the buoyant force on an object always equals the weight of the fluid displaced by the object. This statement is known as Archimedes' principle

❖ Pressure:



Pressure If F is the magnitude of the force exerted on the piston and A is the surface area of the piston, the pressure P of the fluid at the level to which the device has been submerged is defined as the ratio of the force to the area: The SI unit of pressure is the pascal (Pa): $p = \frac{F}{A}$

21-Example: The mattress of a water bed is 2.00 m long by 2.00 m wide and 30.0 cm deep. (A) Find the weight of the water in the mattress. (B) Find the pressure exerted by the water bed on the floor. Solution: (A)

$$V = (2.00\ \text{m})(2.00\ \text{m})(0.300\ \text{m}) = 1.20\ \text{m}^3$$

$$Mg = (1.20 \times 10^3\ \text{kg})(9.80\ \text{m/s}^2) = 1.18 \times 10^4\ \text{N}$$

$$M = \rho V = (1\,000\ \text{kg/m}^3)(1.20\ \text{m}^3) = 1.20 \times 10^3\ \text{kg}$$

(B)

$$P = \frac{1.18 \times 10^4\ \text{N}}{4.00\ \text{m}^2} = 2.94 \times 10^3\ \text{Pa}$$

7th Week

Topic: Properties of Matter:

Bernoulli's Equation

Topic Related Math

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❖ State the Bernoulli's principle:

Bernoulli's equation is a principle in fluid dynamics that describes the conservation of energy in a fluid flow. It states that in a streamline flow, the sum of the pressure energy, kinetic energy, and potential energy per unit volume remains constant.

The equation is given by:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

where:

- P = pressure of the fluid,
- ρ = density of the fluid,
- v = velocity of the fluid,
- g = acceleration due to gravity,
- h = height above a reference point.

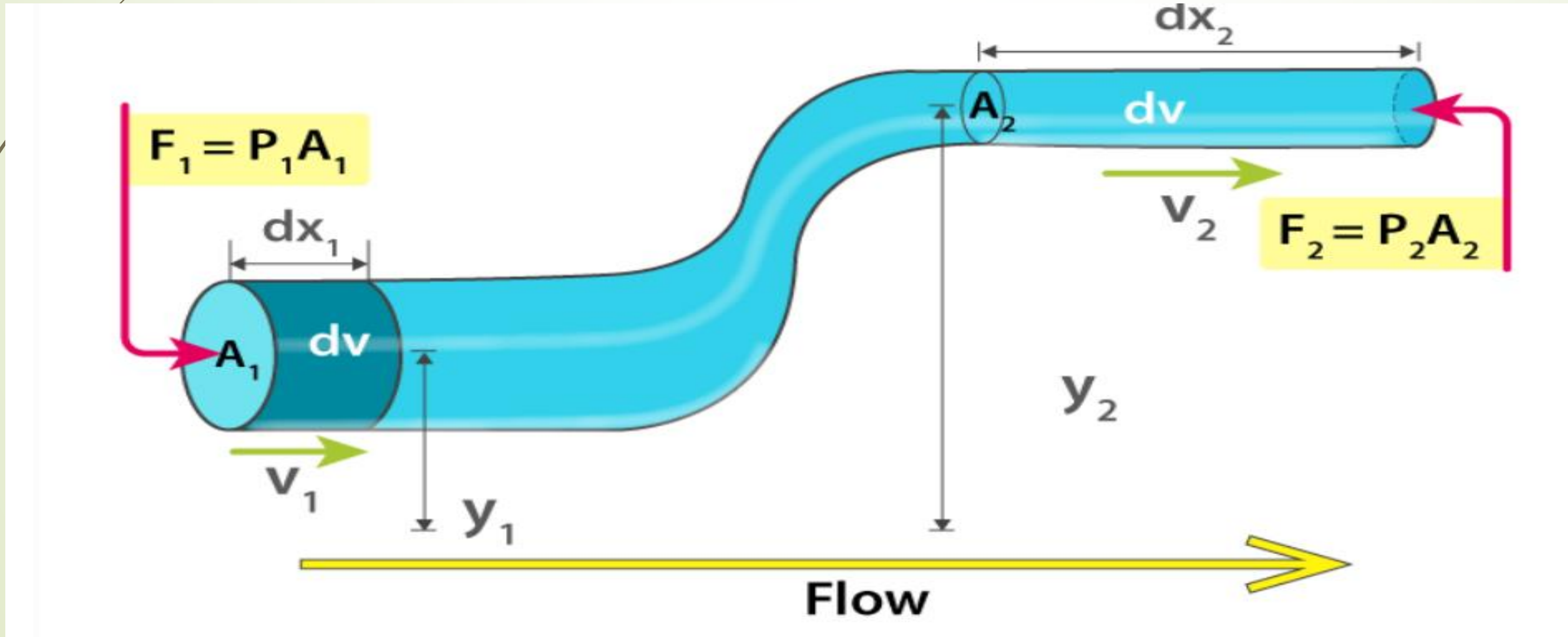
In this equation:

1. P represents the **pressure energy**,
2. $\frac{1}{2}\rho v^2$ represents the **kinetic energy** per unit volume, and
3. $\rho g h$ represents the **potential energy** per unit volume due to height.

Bernoulli's equation applies to incompressible, non-viscous fluids and assumes steady flow along a streamline.

❖ Derivation of Bernoulli's equation :

Consider a pipe with varying diameter and height through which an incompressible fluid is flowing. The relationship between the areas of cross-sections A, the flow speed v, height from the ground y, and pressure p at two different points 1 and 2 are given in the figure below.



Assumptions:

- The density of the incompressible fluid remains constant at both points.
- The energy of the fluid is conserved as there are no viscous forces in the fluid.

Therefore, the work done on the fluid is given as:

$dW = F_1dx_1 - F_2dx_2$

$dW = p_1A_1dx_1 - p_2A_2dx_2$

$dW = p_1dv - p_2dv = (p_1 - p_2)dv$

We know that the work done on the fluid was due to the conservation of change in gravitational potential energy and change in kinetic energy. The change in kinetic energy of the fluid is given as:

$dK = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}\rho dv(v_2^2 - v_1^2)$

The change in potential energy is given as:

$dU = m_2gy_2 - m_1gy_1 = \rho dvg(y_2 - y_1)$

Therefore, the energy equation is given as:

$dW = dK + dU$

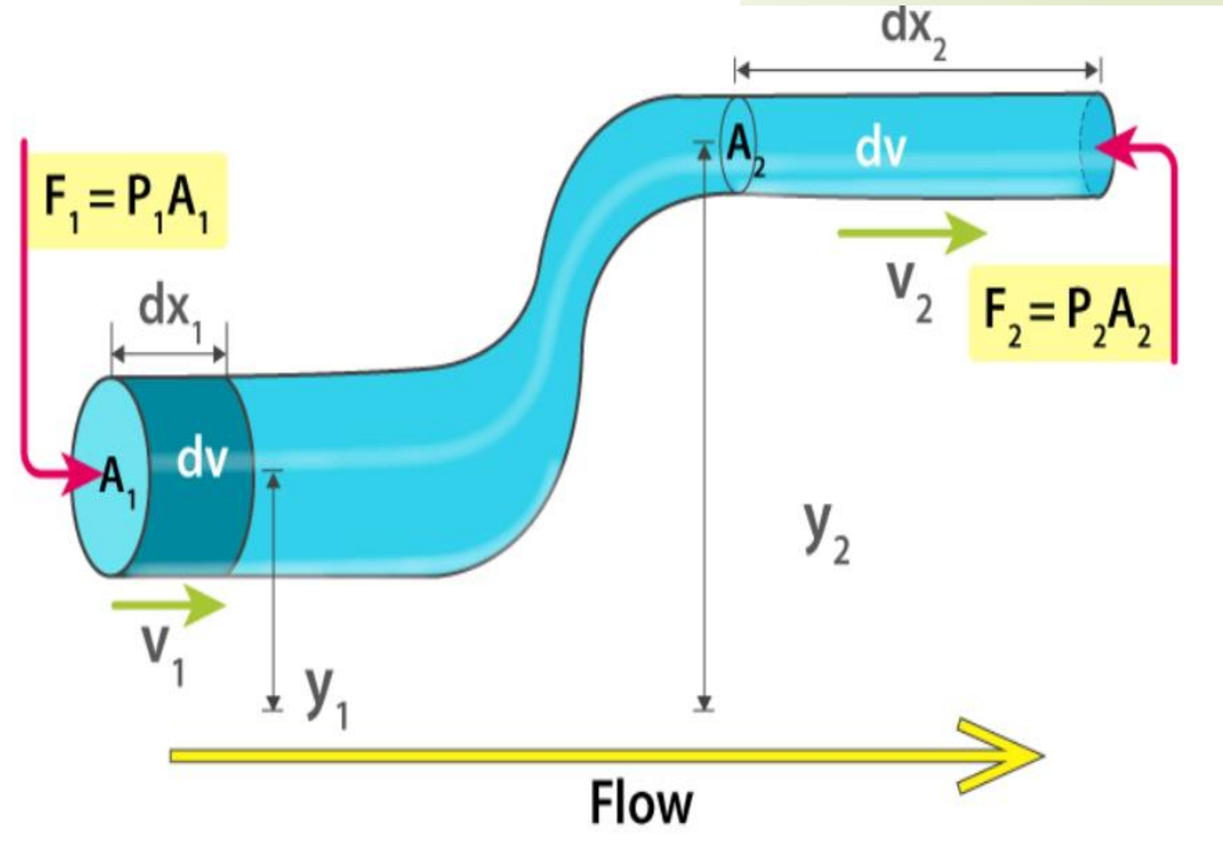
$(p_1 - p_2)dv = \frac{1}{2}\rho dv(v_2^2 - v_1^2) + \rho dvg(y_2 - y_1)$

$p_1 - p_2) = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$

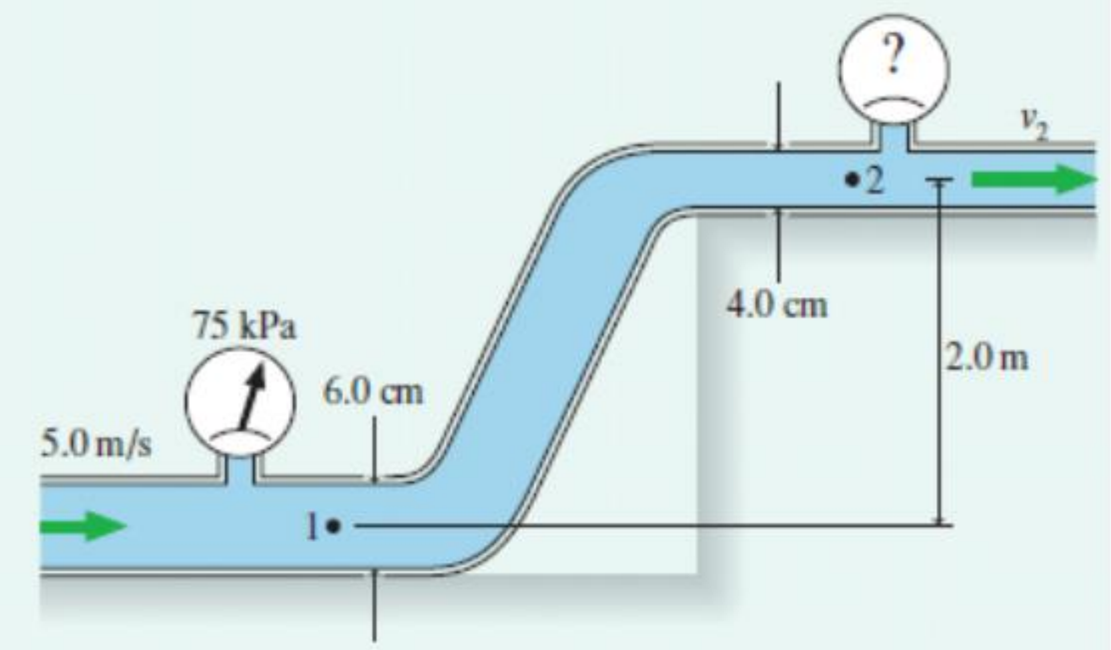
Rearranging the above equation, we get

$p_1 + \frac{1}{2}\rho v_1^2 + \rho gy_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho gy_2$

This is Bernoulli's equation.



Water flows through the pipes shown in FIGURE 15.32. The water's speed through the lower pipe is 5.0 m/s and a pressure gauge reads 75 kPa. What is the reading of the pressure gauge on the upper pipe?



Bernoulli's equation

$$p_2 = p_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 + \rho g y_1 - \rho g y_2$$
$$= p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2)$$

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{A_1}{A_2}v_1 = \frac{r_1^2}{r_2^2}v_1 = \frac{(0.030\text{ m})^2}{(0.020\text{ m})^2}(5.0\text{ m/s}) = 11.25\text{ m/s}$$

The pressure at point 1 is $p_1 = 75\text{ kPa} + 1\text{ atm} = 176{,}300\text{ Pa}$.

We can now use the above expression for *p*2 to calculate:

$p_2 = 105{,}900\text{ Pa}$. $p_2 = 105{,}900\text{ Pa} - 1\text{ atm} = 4.6\text{ kPa}$

Problem 01:

Calculate the pressure in the hose whose absolute pressure is 1.01 x 10^5 $N.m^{-2}$ if the speed of the water in the hose increases from 1.96 $m.s^{-1}$ to 25.5 $m.s^{-1}$. Assume that the flow is frictionless and density 10^3 $kg.m^{-3}$

Problem 02:

Water is flowing in a fire hose with a velocity of 1.0 m/s and a pressure of 200000 Pa. At the nozzle the pressure decreases to atmospheric pressure (101300 Pa), there is no change in height. Use the Bernoulli equation to calculate the velocity of the water exiting the nozzle. (Hint: The density of water is 1000 kg/m^3 and gravity g is 9.8 m/s^2. Pay attention to units!)]

Problem 03:

Through a refinery, fuel ethanol is flowing in a pipe at a velocity of 1 m/s and a pressure of 101300 Pa. The refinery needs the ethanol to be at a pressure of 2 atm (202600 Pa) on a lower level. How far must the pipe drop in height in order to achieve this pressure? Assume the velocity does not change. (Hint: Use the Bernoulli equation. The density of ethanol is 789 kg/m3 and gravity g is 9.8 m/s2. Pay attention to units!)

8th Week

Electricity & Magnetism:

Coulombs law, Band Theory

Topic Related Math

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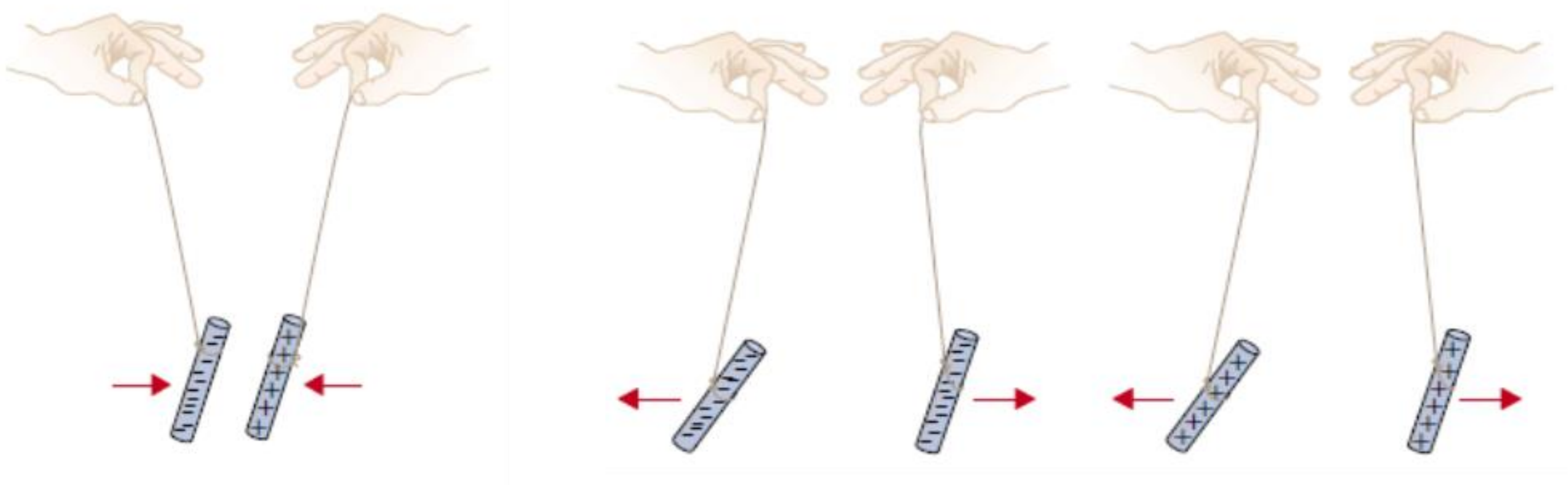
Electricity & Magnetism

Electricity & Magnetism

- Electric Charge,
- Coulomb's law,
- Electric Field,
- Calculation of the Electric Field Strength,
- A dipole in an Electric Field,
- electric Flux and Gauss's Law.
- Electric Potential (V),
- Relation between E & V,
- Electric Potential Energy,
- Capacitor and Capacitance.

Concepts of Electric Charge:

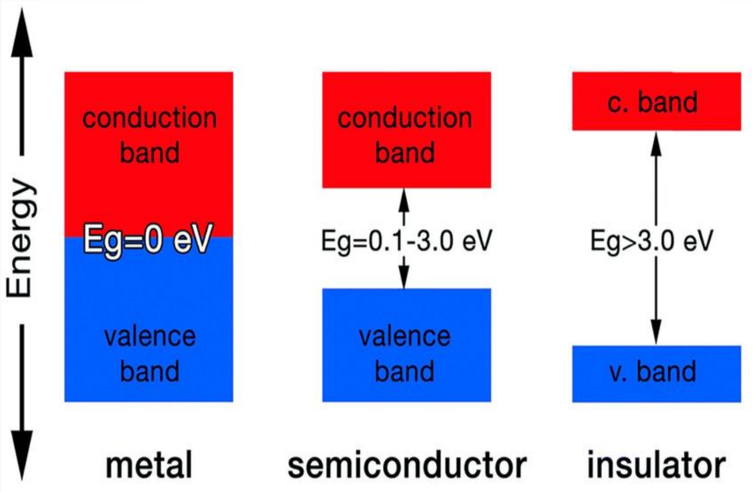


The Laws of Electric Charges:

- Opposite electric charges attract each other.
- Similar electric charges repel each other.
- Charged objects attract some neutral objects.

❖ Describe conductor, semiconductor, and insulator according to band theory

Band theory is a concept in solid-state physics that explains the behavior of electrons in materials based on their energy levels. It categorizes materials into conductors, semiconductors, and insulators based on their electronic band structure.



❑ Conductor:

- ➢ **Band Structure:** In conductors, the valence band and the conduction band overlap, allowing electrons to move freely between them.
- ➢ **Electron Behavior:** The overlap results in a large number of available energy states for electrons, facilitating their movement in response to an electric field.
- ➢ **Conductivity:** Conductors have high electrical conductivity because electrons can easily flow in response to an applied voltage.


Band theory of solids

- **Semiconductor:**

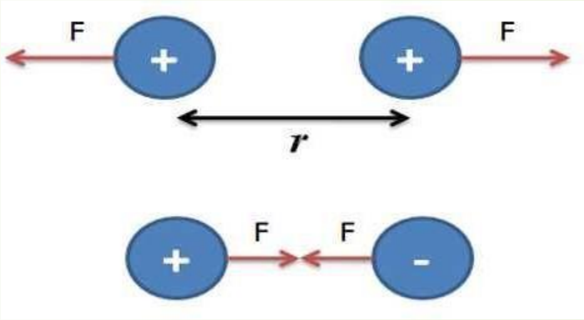
- **Band Structure:** Semiconductors have a small energy gap between the valence band and the conduction band.
- **Electron Behavior:** At absolute zero temperature, semiconductors behave as insulators because all electrons are in the valence band. However, at higher temperatures, some electrons gain enough energy to move to the conduction band.
- **Conductivity:** Semiconductors have moderate electrical conductivity, and their conductivity can be increased significantly by doping with specific impurities or by applying an external voltage.
- An increase in temperature increases the conductivity of a semiconductor because more electrons will have enough energy to move into the conduction band.

- **Insulator:**

- **Band Structure:** The insulator conduction band lacks readily available electrons with a large energy gap between the valence band and the conduction band.
- **Electron Behavior:** At room temperature, or even at elevated temperatures, most electrons are unable to gain enough energy to move into the conduction band. Thus, insulators have very few free-charge carriers.
- **Conductivity:** Insulators have low electrical conductivity due to the lack of readily available electrons in the conduction band.

❖ Coulomb's Law:

Coulomb's law states that the electrical force between two charged objects is directly proportional to the product of the quantity of charge on the objects and inversely proportional to the square of the separation distance between the two charges. In equation form, Coulomb's law can be written as



$$F = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1 q_2}{d^2}$$

- where q_1 represents the quantity of charge on object 1 (in Coulombs), q_2 represents the quantity of charge on object 2 (in Coulombs), and d represents the distance of separation between the two objects (in meters).
- The symbol $\frac{1}{4\pi\varepsilon_\circ}$ is a proportionality constant known as the Coulomb's law constant. The value of this constant is dependent upon the medium that the charged objects are immersed in. In the case of air, the value is approximately 9.0 x 10^9 N • m^2 / C^2.
- The force acting between similar charge is repulsive and the force acting between opposite charge is attractive.

Problem-22:

What is the magnitude of the force of repulsion between two small spheres 1.0 m apart, if each has a charge of 1.0×10^{-12} C?

Solution

$q_1 = q_2 = 1.0 \times 10^{-12}$ C

$r = 1.0$ m

$F_E = ?$

$$F_E = \frac{kq_1q_2}{r^2}$$

$$= \frac{(9.0 \times 10^9\ \text{N·m}^2/\text{C}^2)(1.0 \times 10^{-12}\ \text{C})^2}{(1.0\ \text{m})^2}$$

$$F_E = 9.0 \times 10^{-15}\ \text{N}$$

The magnitude of the force of repulsion is 9.0×10^{-15} N, a very small force.

❑ What Is an Electric Field?

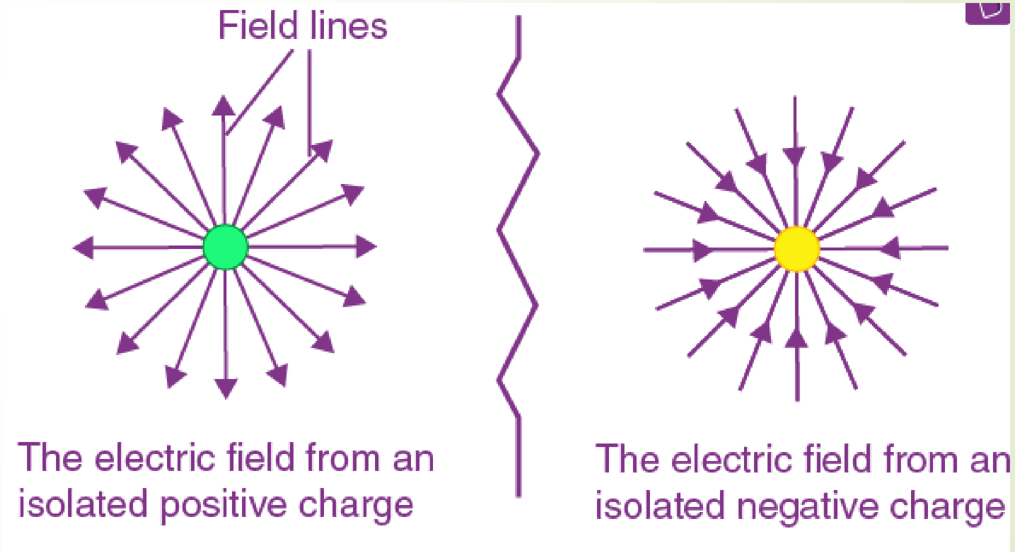
An electric field is defined mathematically as a vector field that can be associated with each point in space, the force per unit charge exerted on a positive test charge at rest at that point.

The formula of the electric field is given as,

$E = F/Q$

Where,

E is the electric field. F is the force. Q is the charge.



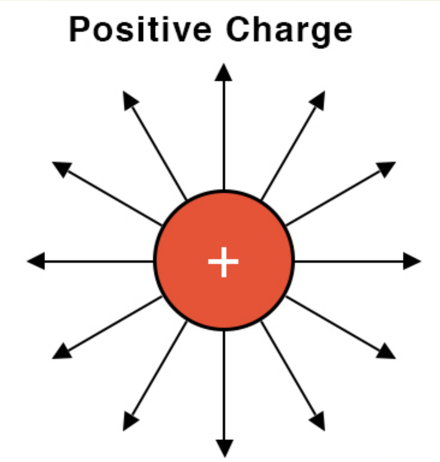
The direction of the field is taken as the direction of the force which is exerted on the positive charge. The electric field is radially outwards from the positive charge and radially towards the negative point charge.

❖ Explaining Electric field lines for a positive, negative, positive-positive, positive-negative, and negative-negative charge

Electric field lines for a positive charge

1.Originating from the Charge: Electric field lines for a positive charge originate from the charge itself. They extend radially outward in all directions, spreading out like spokes on a wheel.

2.Pointing Away from the Charge: These field lines point away from the positive charge. This indicates the direction a positive test charge placed at any point would experience a force if it were free to move. In simpler terms, positive charges would be repelled if they were nearby.



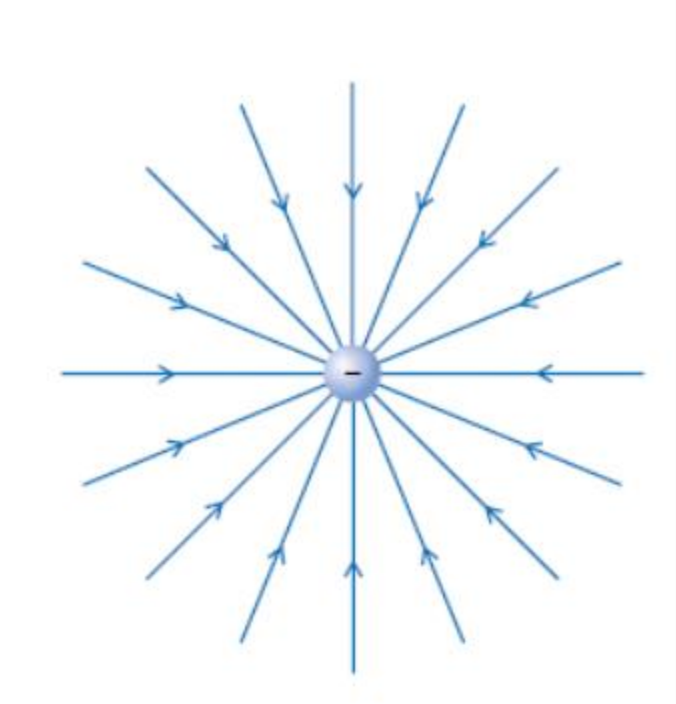
3.Strength of the Field: The density of the field lines represents the strength of the electric field. Near the positive charge, where the field is strongest, the lines are closer together. As you move farther away, the lines become more spaced out, indicating weaker field strength.

4.Never Intersecting: Electric field lines never intersect. This means that at any point in space, the direction of the electric field is well-defined.

5.Obeying the Inverse Square Law: The intensity of the electric field weakens as you move away from the positive charge, following the inverse square law. This means that the strength of the electric field diminishes with the square of the distance from the charge.

Explaining Electric field lines for a negative charge

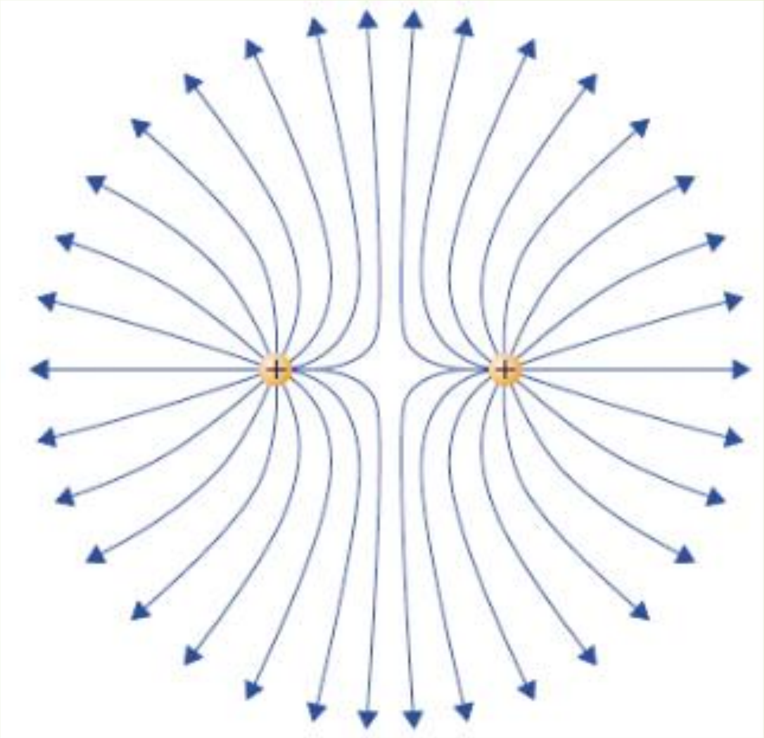
1. **Originating from the Charge:** Similar to a positive charge, electric field lines for a negative charge also originate from the charge itself. However, they extend radially inward in all directions, rather than outward.
2. **Pointing Towards the Charge:** The field lines around a negative charge point inward, towards the charge. This indicates that a positive test charge placed at any point would be attracted towards the negative charge.
3. **Strength of the Field:** Just like with a positive charge, the density of the field lines represents the strength of the electric field. Near the negative charge, where the field is strongest, the lines are closer together. As you move farther away, the lines become more spaced out, indicating weaker field strength.
4. **Never Intersecting:** Electric field lines around a negative charge, like with a positive charge, never intersect. This means that at any point in space, the direction of the electric field is well-defined.



Negatively charged sphere

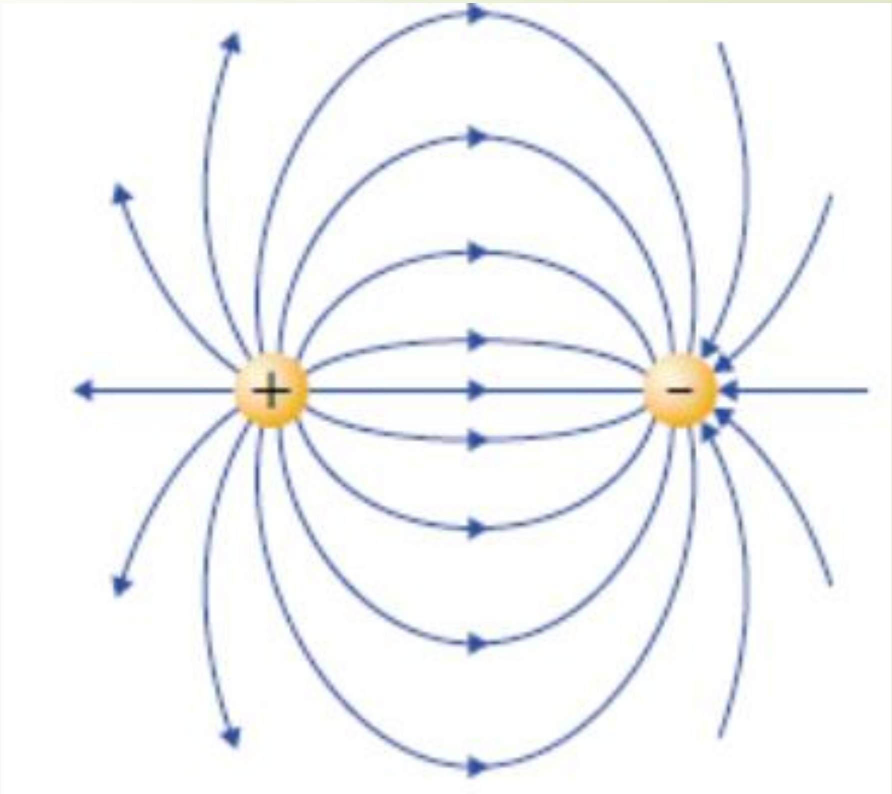
Explaining Electric field lines for a positive-positive charge

1. **Originating from Each Charge:** Electric field lines originate from each positive charge. They extend radially outward from each charge, just as they would for a single positive charge.
2. **Repulsion Between Field Lines:** Since like charges repel each other, the electric field lines from each positive charge repel each other as well. This results in the field lines diverging as they move away from each charge.
3. **Overall Field Pattern:** The overall pattern of electric field lines resembles that of two individual positive charges, with no intersection between their respective field lines. The lines are denser near each charge and become less dense as you move away from them.
4. **Direction of the Field:** At any point between the two charges, the electric field lines indicate the direction a positive test charge would experience a force if placed there. The direction of the force would be away from both charges, due to their repulsion.



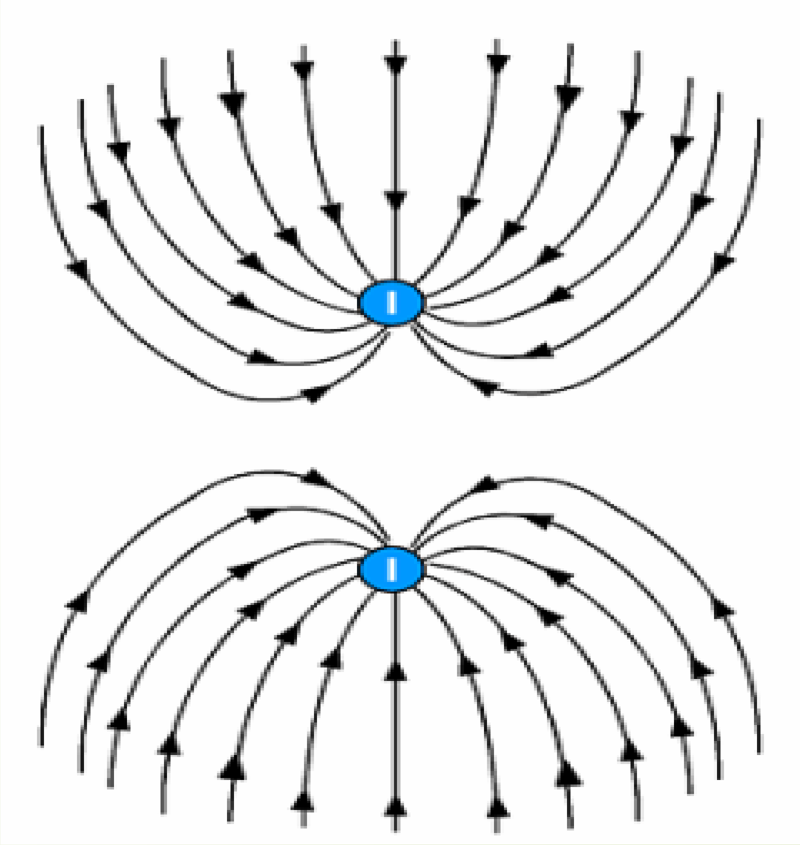
Electric field lines for a positive-negative charge

1. **Originating from Each Charge:** Electric field lines originate from each charge. The positive charge emits lines that extend radially outward, while the negative charge emits lines that extend radially inward.
2. **Convergence of Field Lines:** Due to the attraction between opposite charges, the electric field lines from the positive and negative charges converge towards each other. This is in contrast to the divergence seen with like charges.
3. **Direction of the Field:** Between the positive and negative charges, the electric field lines indicate the direction a positive test charge would experience a force if placed there. The direction of the force would be towards the negative charge and away from the positive charge, due to the attraction and repulsion between opposite charges.



Electric field lines for a negative-negative charge

1. **Originating from Each Charge:** Electric field lines originate from each negative charge. Similar to a single negative charge, these lines extend radially inward in all directions.
2. **Repulsion Between Field Lines:** Since like charges repel each other, the electric field lines from each negative charge also repel each other. This results in the field lines diverging as they move away from each charge.
3. **Overall Field Pattern:** The overall pattern of electric field lines resembles that of two individual negative charges, with no intersection between their respective field lines. The lines are denser near each charge and become less dense as you move away from them.
4. **Direction of the Field:** At any point between the two charges, the electric field lines indicate the direction a negative test charge would experience a force if placed there. The direction of the force would be away from both charges, due to their repulsion.



Problem-23

What is the electric field 0.60 m away from a small sphere with a

A charge of 1.2×10^{-8}C

Solution:

$$r = 0.60\ m$$

$$E = ?$$

$$E = \frac{kq}{r^2} = \frac{\left(9.0 \times 10^9 N.\frac{m^2}{2}\right)(1.2 \times 10^{-8} C)}{(0.60\ m)^2}$$

$$= 3.0 \times 10^2 N/C$$

$\vec{E} = 3.0 \times 10^{2} N/C$ [radially outward]

9th Week

Topic: Electricity & Magnetism:

Electric potential Energy,

Topic Related Math

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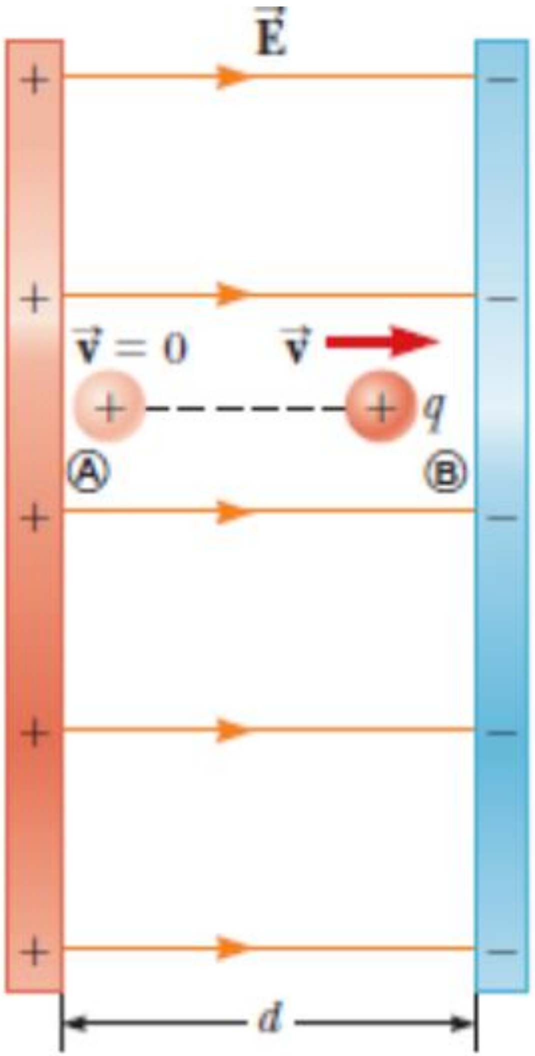
Motion of a Charged Particle in a Uniform Electric Field

When a particle of charge q and mass m is placed in an electric field $\vec{E}$, the electric force exerted on the charge is $q\vec{E}$. If that is the only force exerted on the particle, it must be the net

according to the particle under a net force $F \quad m\vec{a}$). Therefore,

$$F_E \quad q \quad \vec{E} = m\vec{a}$$

$$\vec{a} = q\vec{E}/m$$



Example

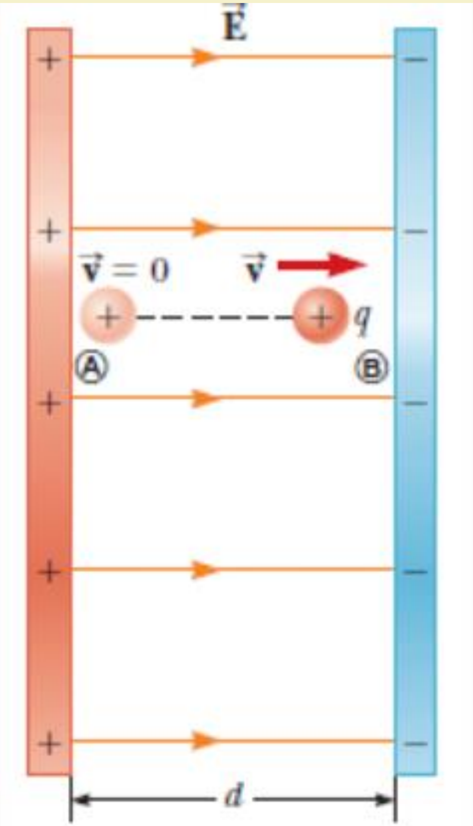
A uniform electric field $\vec{E}$ is directed along the x axis between parallel plates of charge separated by a distance d as shown in Figure 23.23. A positive point charge q of mass m is released from rest at a point Ⓐ next to the positive plate and accelerates to a point Ⓑ next to the negative plate.

(A) Find the speed of the particle at Ⓑ by modeling it as a particle under constant acceleration.

Solution:

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = 0 + 2a(d - 0) = 2ad$$

$$v_f = \sqrt{2ad} = \sqrt{2\left(\frac{qE}{m}\right)d} = \sqrt{\frac{2qEd}{m}}$$



Example

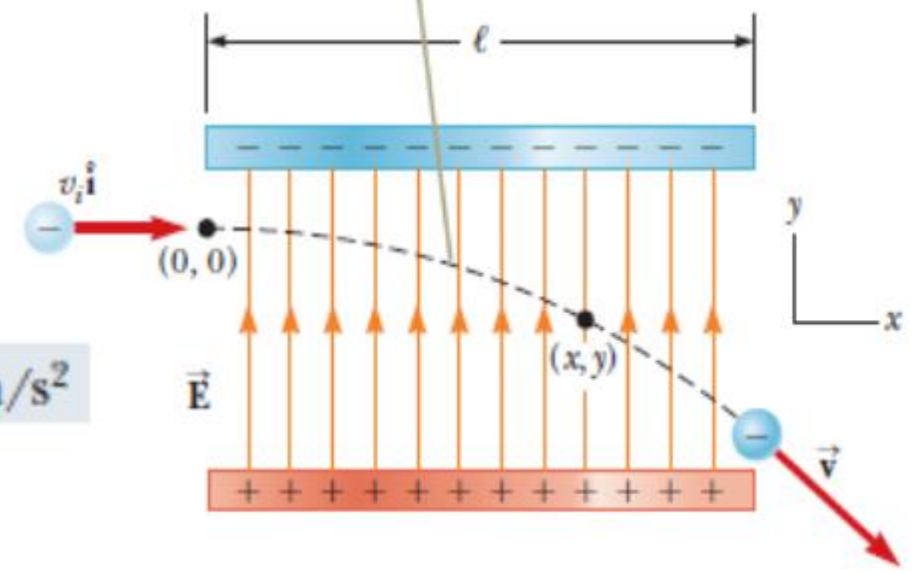
An electron enters the region of a uniform electric field as shown in Figure 23.24, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m.

(A) Find the acceleration of the electron while it is in the electric field.

$$a_y = -\frac{eE}{m_e}$$

$$a_y = -\frac{(1.60 \times 10^{-19}\ \text{C})(200\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}} = -3.51 \times 10^{13}\ \text{m/s}^2$$

The electron undergoes a downward acceleration (opposite $\vec{E}$), and its motion is parabolic while it is between the plates.

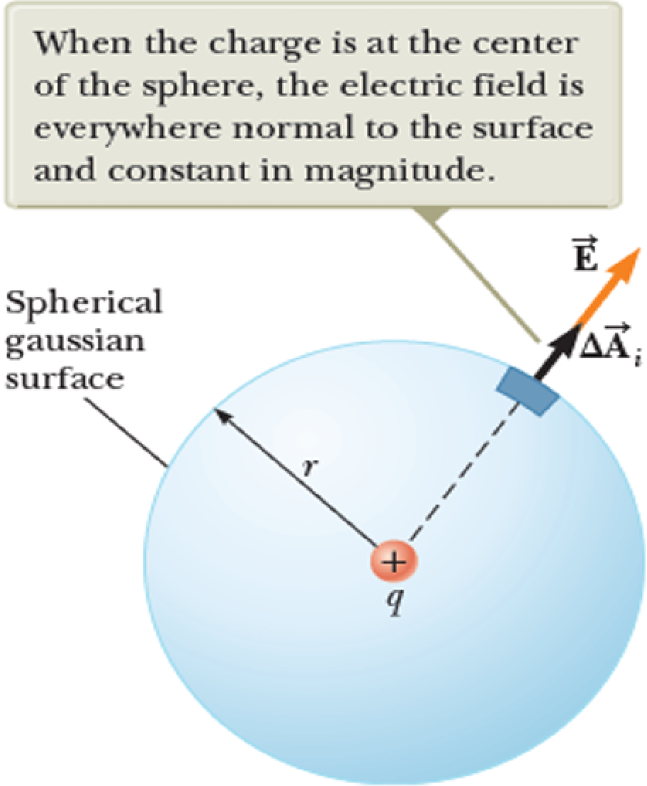


Gauss's Law

A general relationship between the net electric flux through a closed surface and the charge enclosed by the surface is known as *Gauss's law.*

The magnitude of the electric field everywhere on the surface of the sphere is $E = k_e q/r^2$.

The net flux through the gaussian surface is



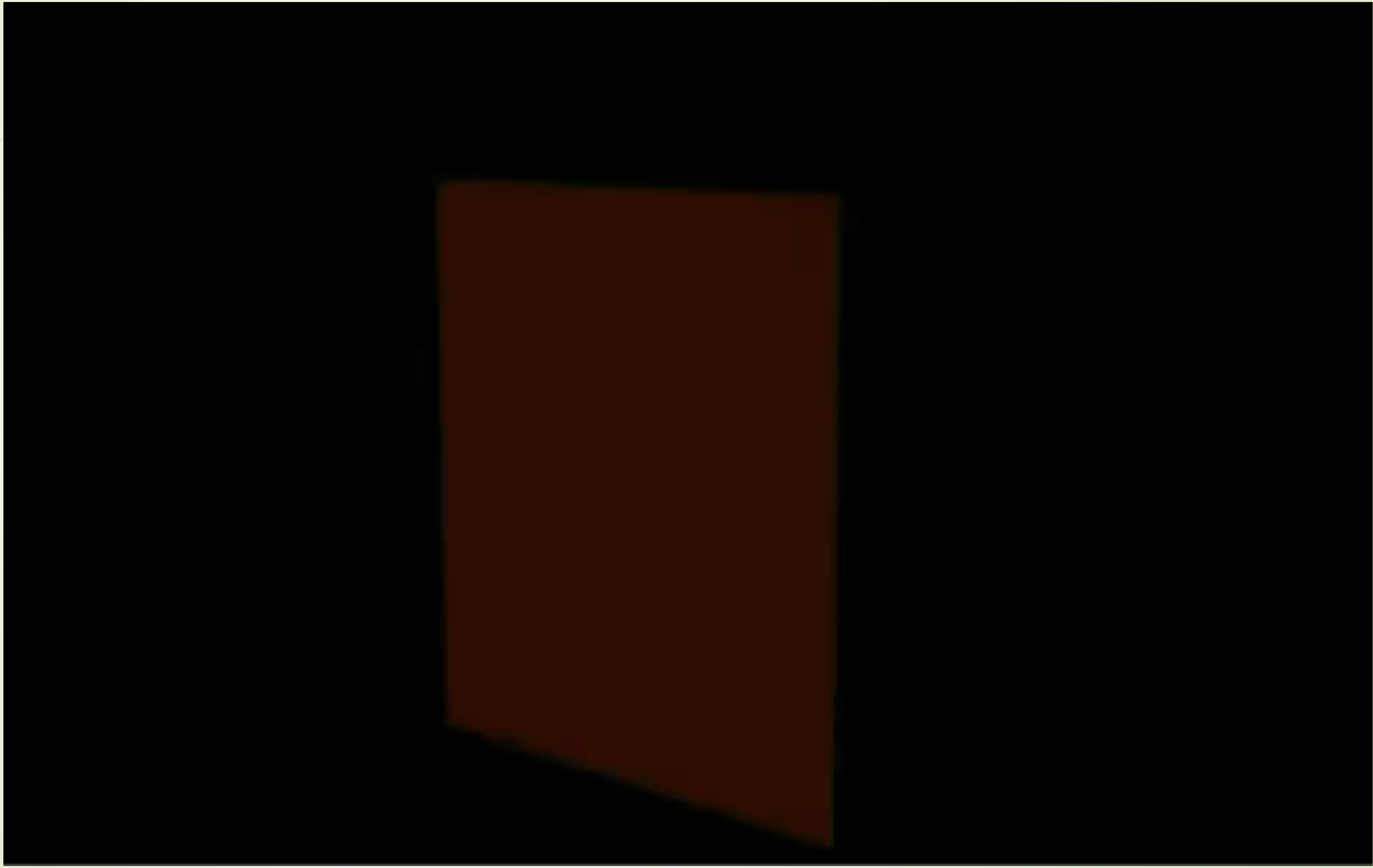
$$\Phi_E = \oint \vec{E}.d\vec{A} = \oint EdA = E\oint dA = \left(\frac{k_e q}{r^2}\right)(4\pi r^2)$$

$$= 4\pi k_e q = \frac{q}{\epsilon_0}$$

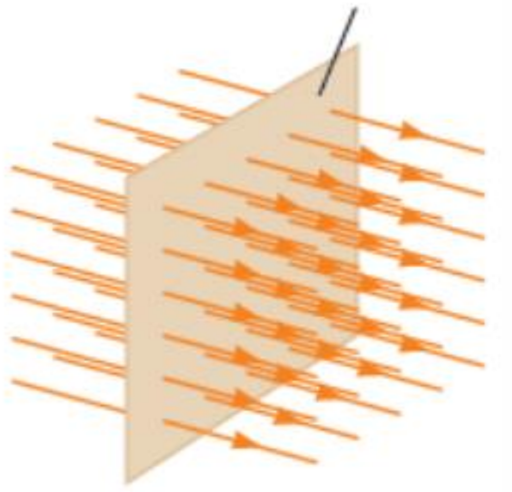
Gauss's law says that the net electric flux Φ_E through any closed gaussian surface is equal to the *net* charge q_{in} inside the surface divided by ϵ_0:

$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

Simulation of Gauss's Law

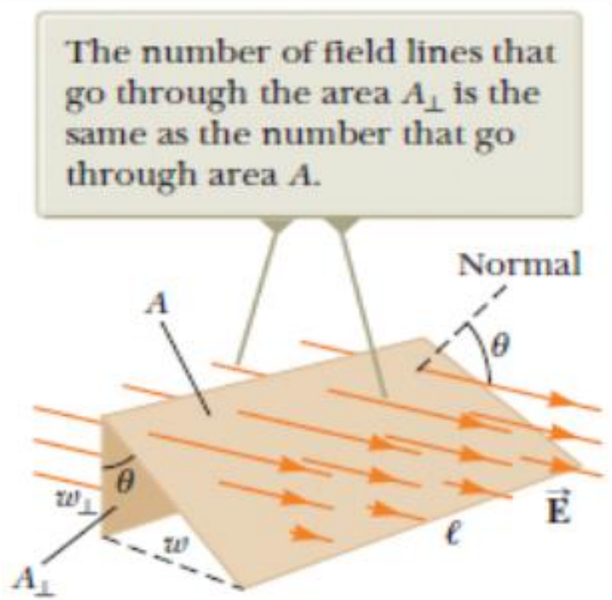


Electric Flux:



Uniform electric field penetrating a

perpendicular to the field.



Uniform electric field penetrating an area *A* whose normal is at an angle $\theta$to the

The total number of lines penetrating the surface is proportional to the product *EA.* This product of the magnitude of the electric field and surface area perpendicular to the field is called the **electric flux**:

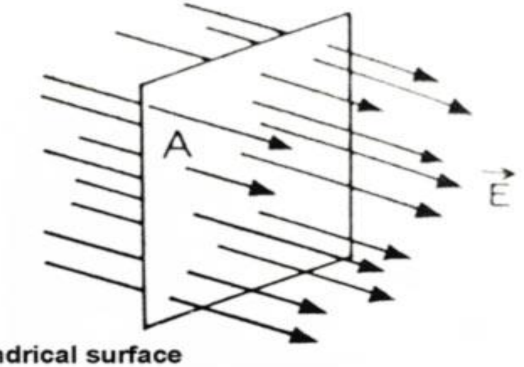
Electric Flux Formula

$$\Phi_E = E\,A\cos\theta\,.$$

When θ = 0

$$\Phi_E = EA\,.$$

E = electric field
A = area of the surface
E = magnitude
θ = the angle between the electric field lines and the normal (perpendicular) to S
Φ = flux of electric field through a closed cylindrical surface



Flux through a *closed surface*

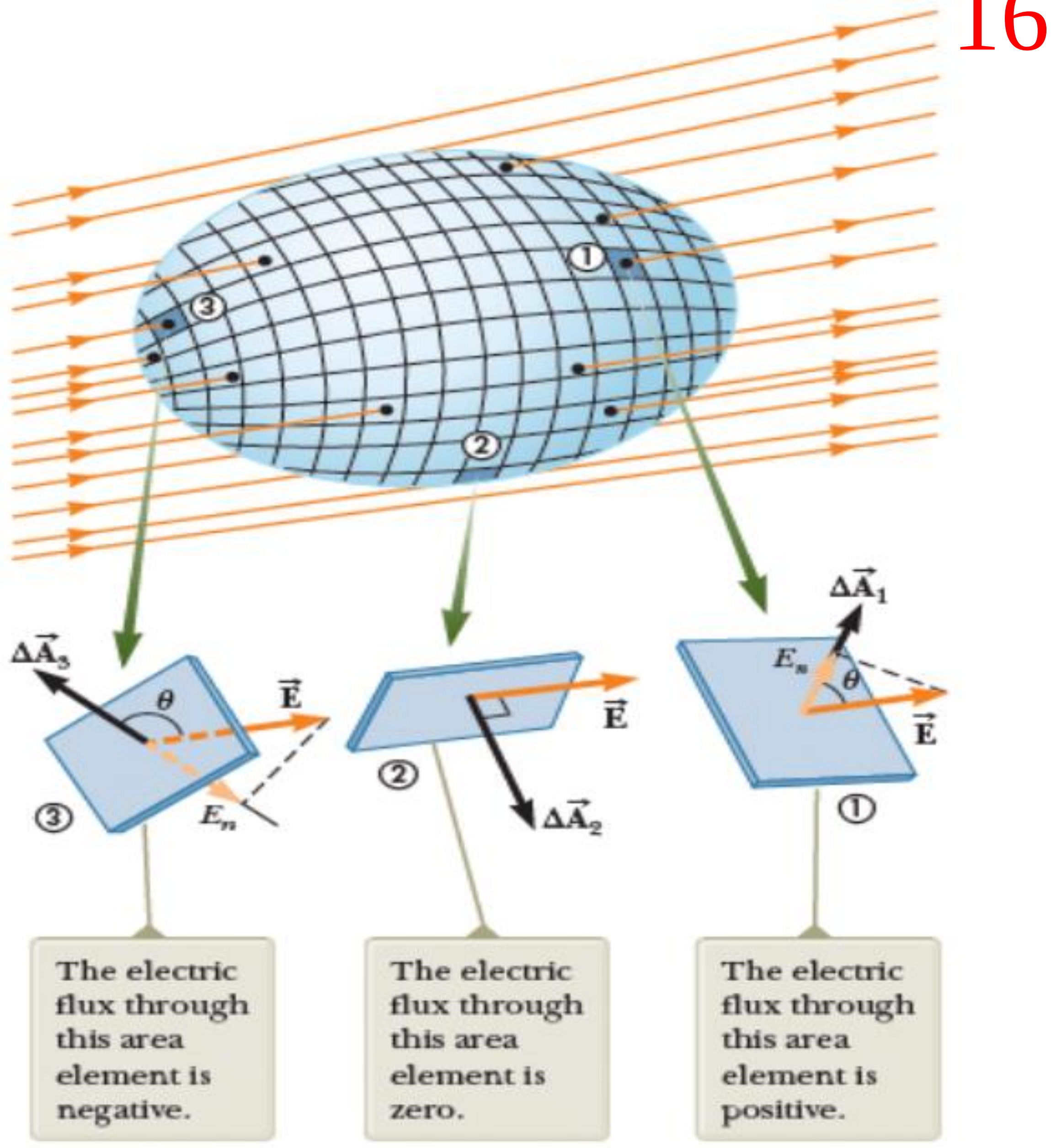
The *net* flux through the surface is

leaving the surface, where the net number means *the number of lines*

lines entering the surface. If more lines are leaving than entering, the net flux is positive. If more lines are entering than leaving, the net flux is negative.

$$\Phi_E = \oint \vec{E}.d\vec{A} = \oint E_n dA$$

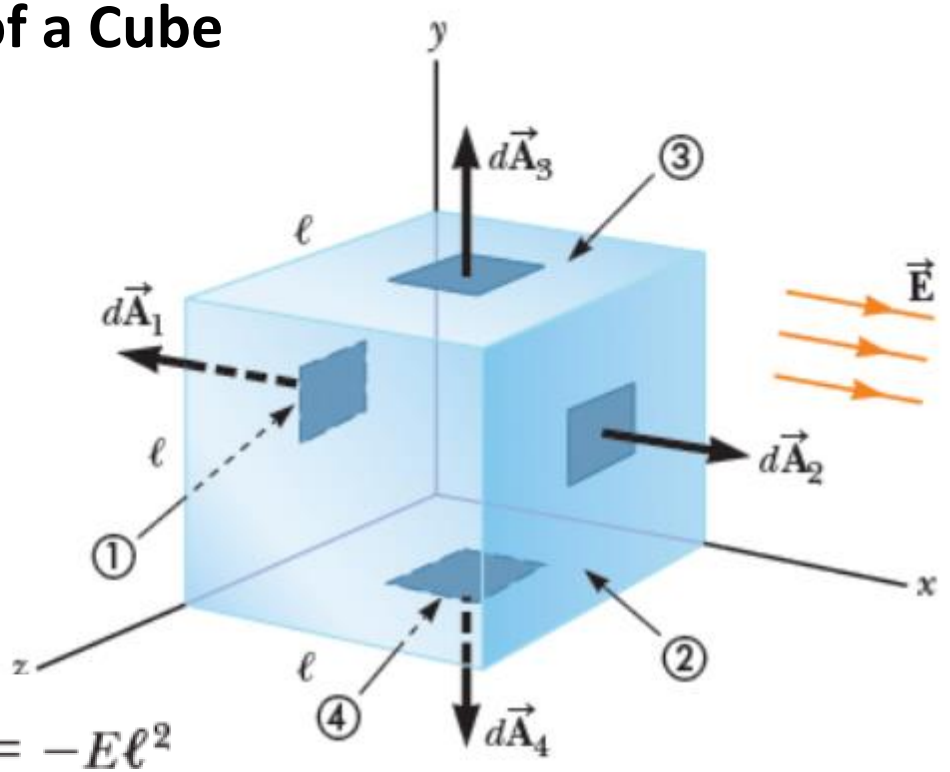
where E_n is the component normal to the surface.



Problem-24: Determine the Total electric flux of a Cube

Consider a uniform electric field $\vec{E}$ oriented in the x direction in empty space. A cube of

as shown in Fig. Find the net electric flux through the surface of the cube.



$$\Phi_E = \int_1 \vec{E} \cdot d\vec{A} + \int_2 \vec{E} \cdot d\vec{A}$$

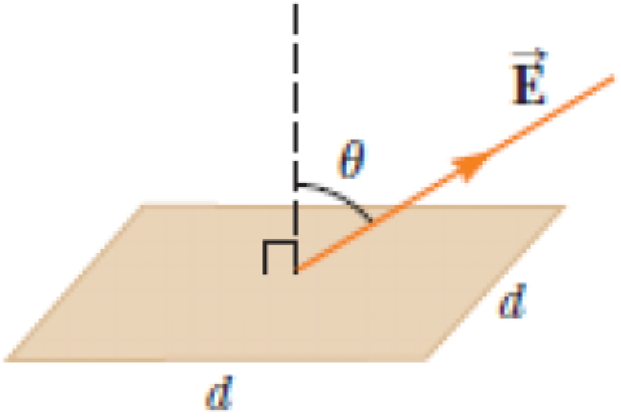
$$\int_1 \vec{E} \cdot d\vec{A} = \int_1 E(\cos 180°)\, dA = -E\int_1 dA = -EA = -E\ell^2$$

$$\int_2 \vec{E} \cdot d\vec{A} = \int_2 E(\cos 0°)\, dA = E\int_2 dA = +EA = E\ell^2$$

Find the net flux by adding the flux over all six faces:

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = 0$$

Assignment:



1. Consider a plane surface in a uniform electric field as in Fig., where $d=$ 0 2 C, find the magnitude of the electric field.
2. Find the electric flux through the plane surface shown in Fig. if $\theta = 60^0$, entire area of the surface.

❑ Deduce Coulomb's law from Gauss's law

From Gauss's Law

$$\oint E \cdot da = E \oint da = \frac{q}{\varepsilon_0}$$

Since E is constant at all points on the surface,

$$EA = \frac{q}{\varepsilon_0}$$

Surface area of sphere is $A = 4\pi r^2$ so,

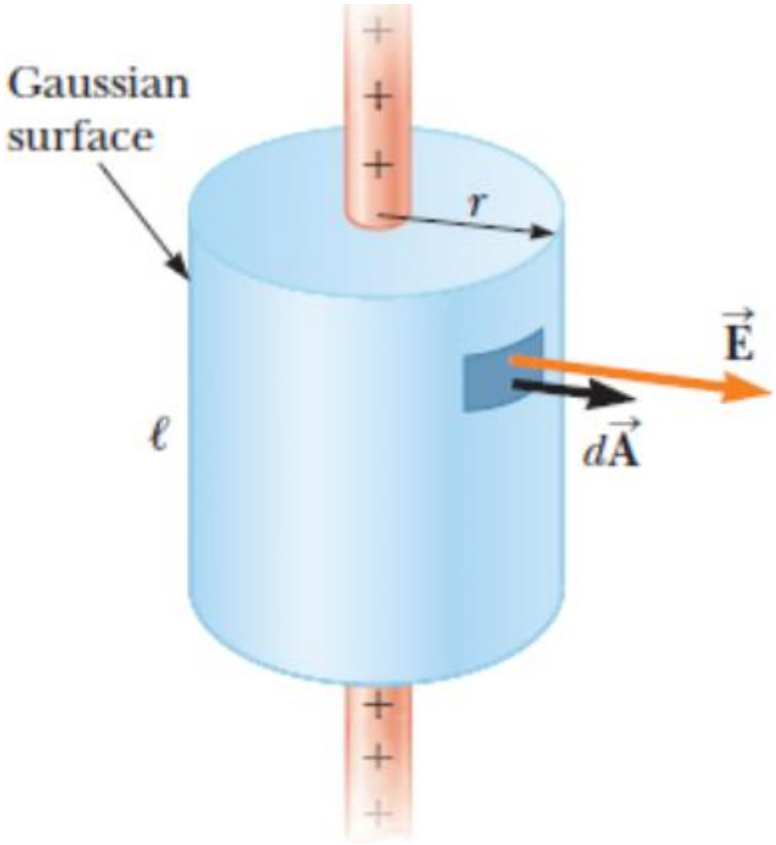
$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

For a point charge q' at a distance r from charge q force would be $F = q'E$ or,

$$F = \frac{1}{4\pi\varepsilon_0} \frac{qq'}{r^2}$$

❖ Determine the electric field for a cylindrical symmetric charge distribution

Find the electric field a distance *r* from a line of positive charge of infinite length



Solution:

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E \oint dA = EA = \frac{q_{in}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0}$$

Substitute the area $A = 2\pi r \ell$

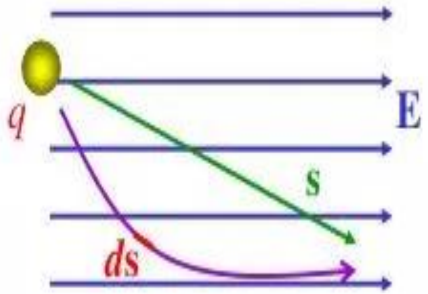
$$E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi \epsilon_0 r} = 2k_e \frac{\lambda}{r}$$

Electric Potential

Electric Energy

- Electric fields produce forces; forces do work
- Since the electric fields are doing work, they must have potential energy
  - The amount of work done is the change in the potential energy
- The force can be calculated from the charge and the electric field



$$W = \mathbf{F} \cdot \mathbf{s}$$

$$\Delta U = -W = -\mathbf{F} \cdot \mathbf{s}$$

$$\mathbf{F} = q\mathbf{E}$$

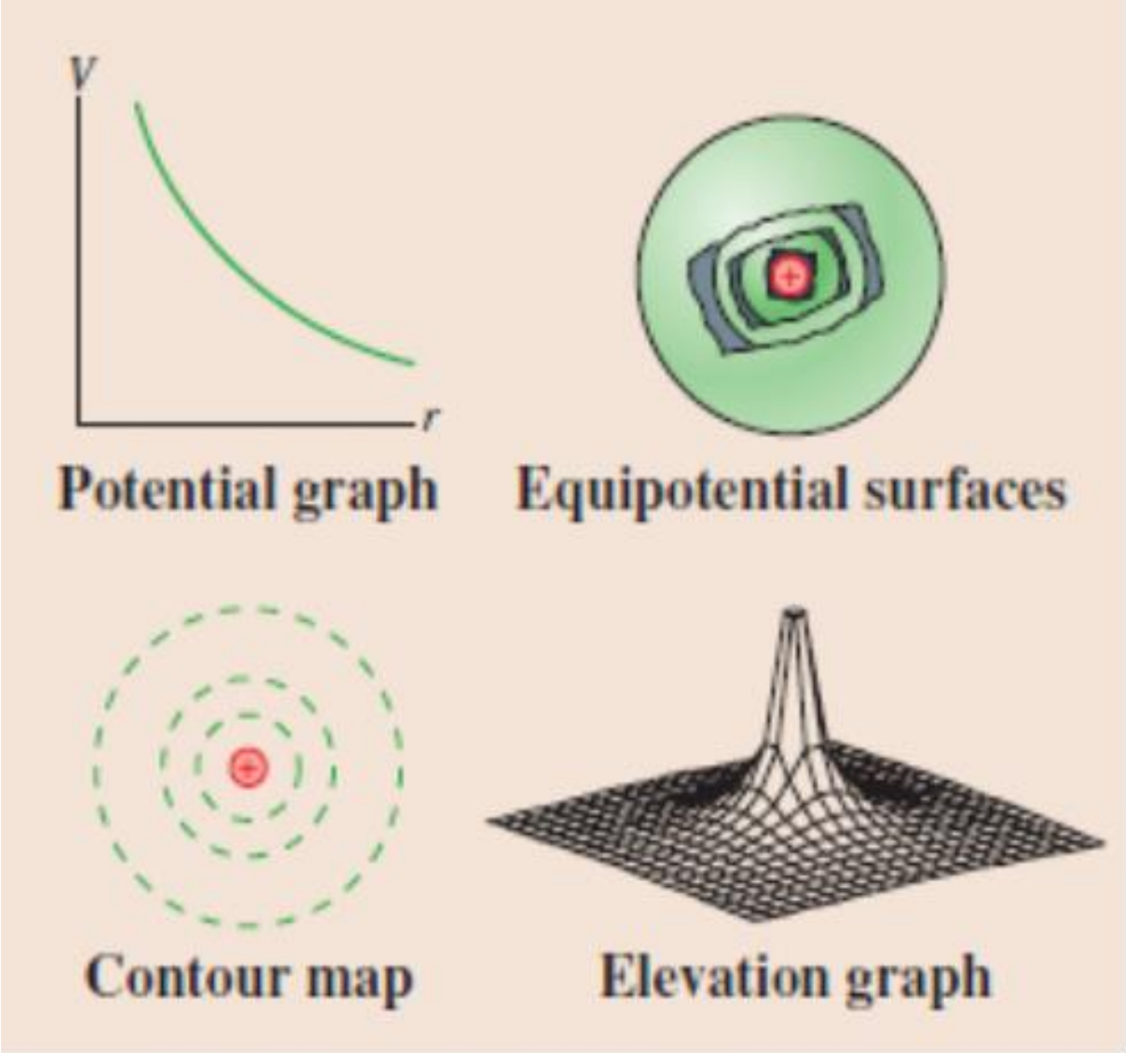
$$\Delta U = -q\mathbf{E} \cdot \mathbf{s}$$

- If the path or the electric field are not straight lines, we can get the change in energy by integration
  - Divide it into little steps of size *ds*
  - Add up all the little steps

$$dU = -q\mathbf{E} \cdot d\mathbf{s}$$

$$\Delta U = -q\int \mathbf{E} \cdot d\mathbf{s}$$

Graphical representations of the potential:



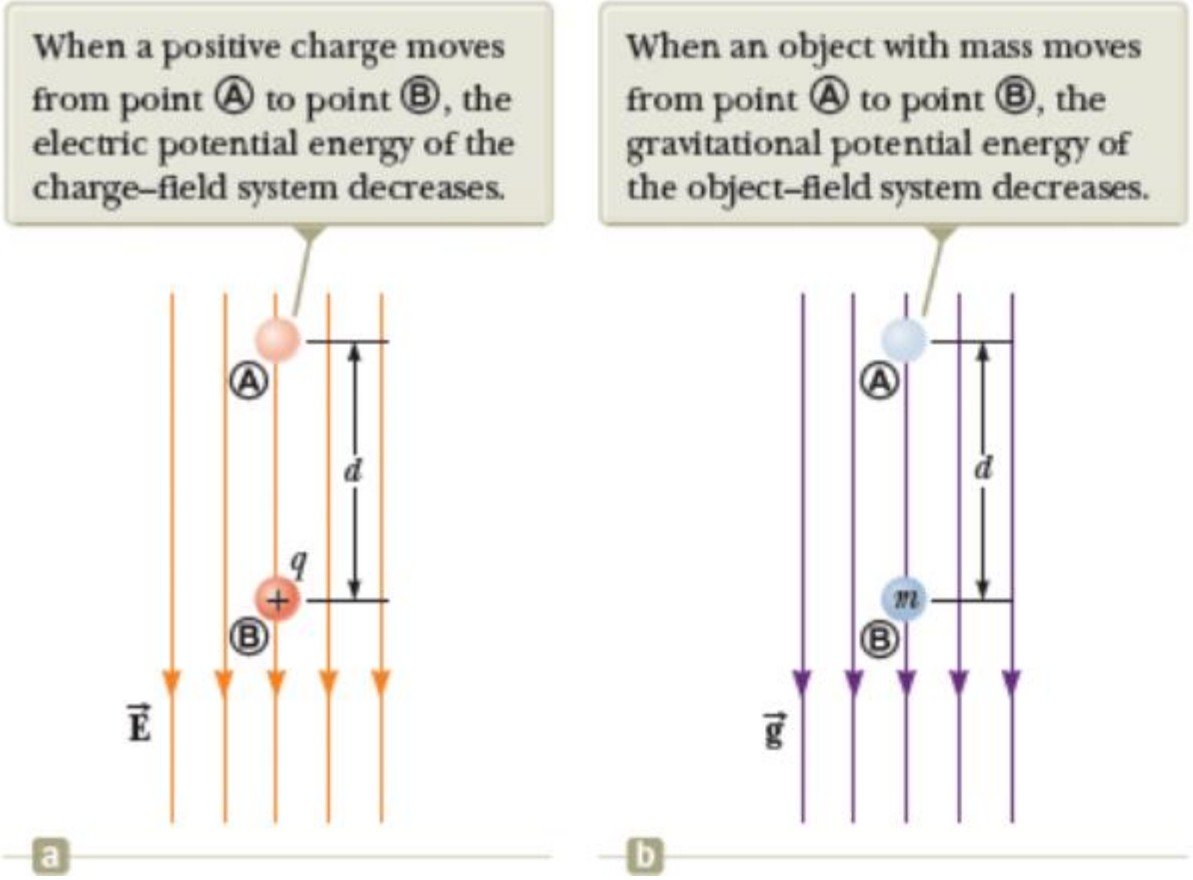
Potential Difference in a Uniform Electric Field

(a) When the electric field $\vec{E}$ is directed downward, point B is at a lower electric potential than point A. (b) A gravitational analog to the situation in (a).

- Potential difference between two points in a uniform electric field

  B A

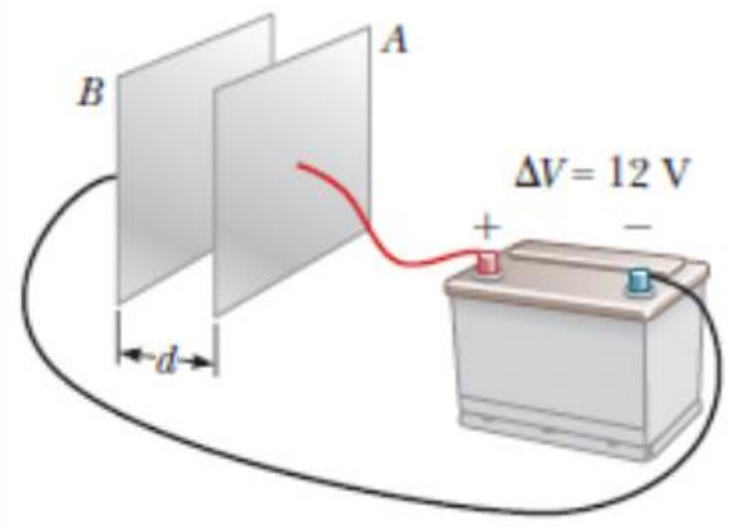
- The potential energy of the charge–field system



Example The Electric Field Between Two Parallel Plates of Opposite Charge

A battery has a specified potential difference its terminals and establishes that

the terminals. A 12-V battery is connected between two parallel plates as shown in Fig. The separation between the plates is d = 0.30 cm, and we assume the electric field between the plates to be uniform. Find the magnitude of the electric field between the plates.



Solution:

$$E = \frac{|V_B - V_A|}{d} = \frac{12\text{ V}}{0.30 \times 10^{-2}\text{ m}} = 4.0 \times 10^{3}\text{ V/m}$$

Example Motion of a Proton in a Uniform Electric Field

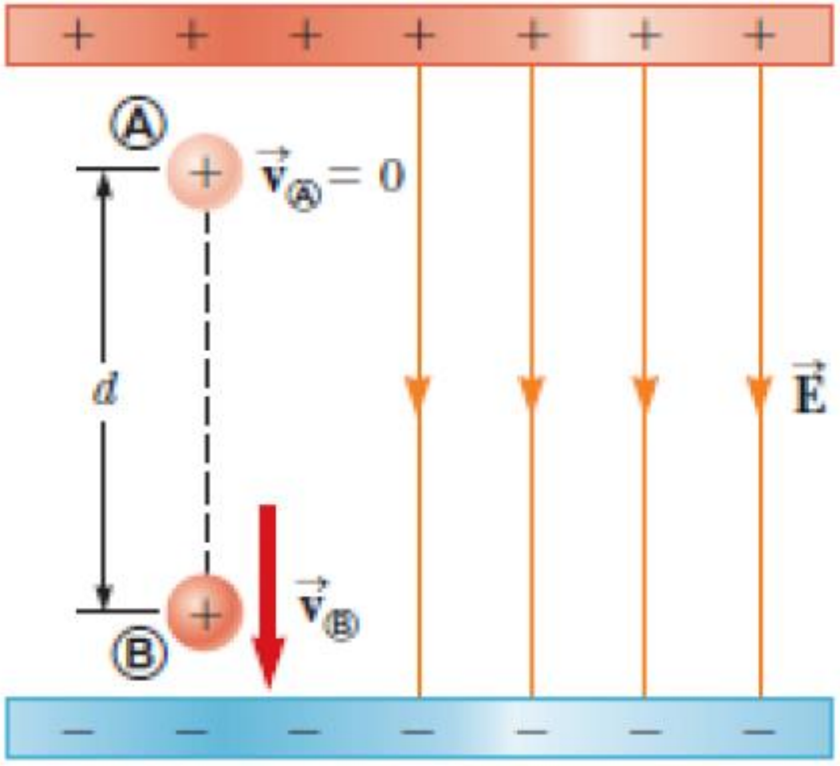
A proton is released from rest at point A in a uniform electric field that has a magnitude of ^{4} V/m. The proton undergoes a

in the direction of $\vec{E}$. Find the speed of the proton after completing the displacement. **Solution:**

$$\Delta K + \Delta U = 0 \quad \text{or} \quad \left(\tfrac{1}{2}mv^2 - 0\right) + e\,\Delta V = 0$$

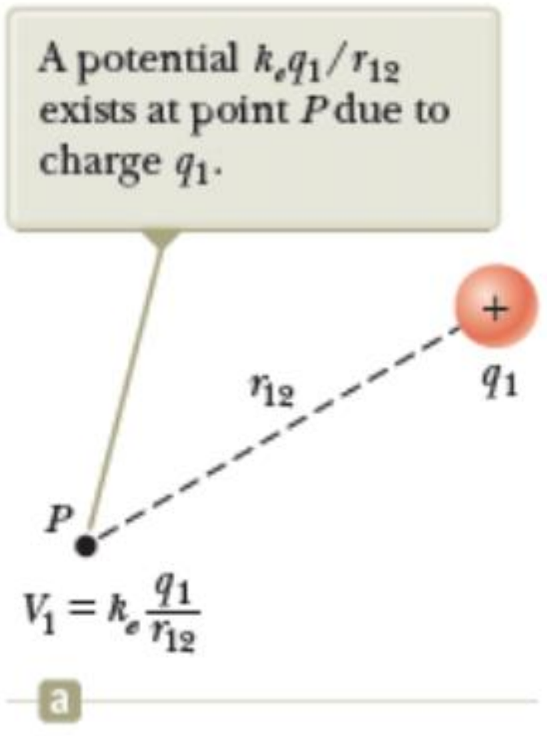
$$v = \sqrt{\frac{-2e\,\Delta V}{m}} = \sqrt{\frac{-2e(-Ed)}{m}} = \sqrt{\frac{2eEd}{m}}$$

$$v = \sqrt{\frac{2(1.6 \times 10^{-19}\,\text{C})(8.0 \times 10^{4}\,\text{V})(0.50\,\text{m})}{1.67 \times 10^{-27}\,\text{kg}}} = 2.8 \times 10^{6}\,\text{m/s}$$



Electric potential & Electric potential energy: (a) Charge q_1 establishes an electric

1



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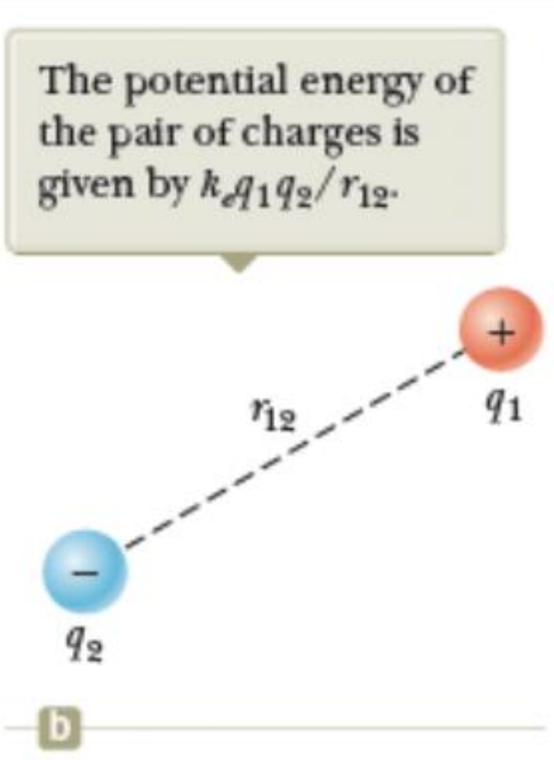
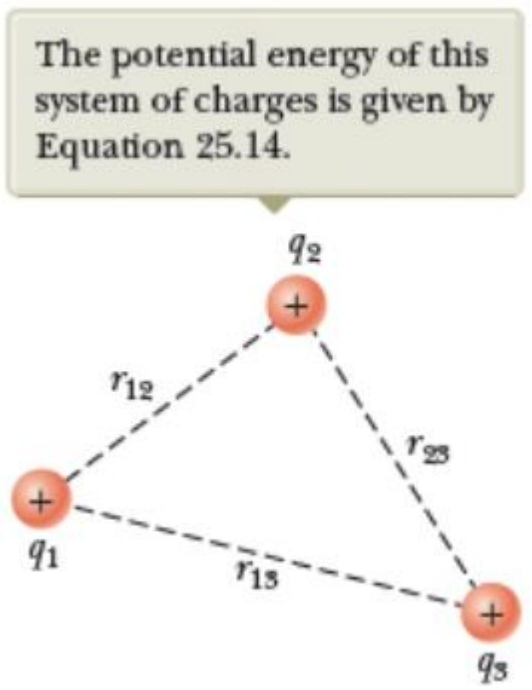




$$V = k_e \sum_i \frac{q_i}{r_i}$$

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

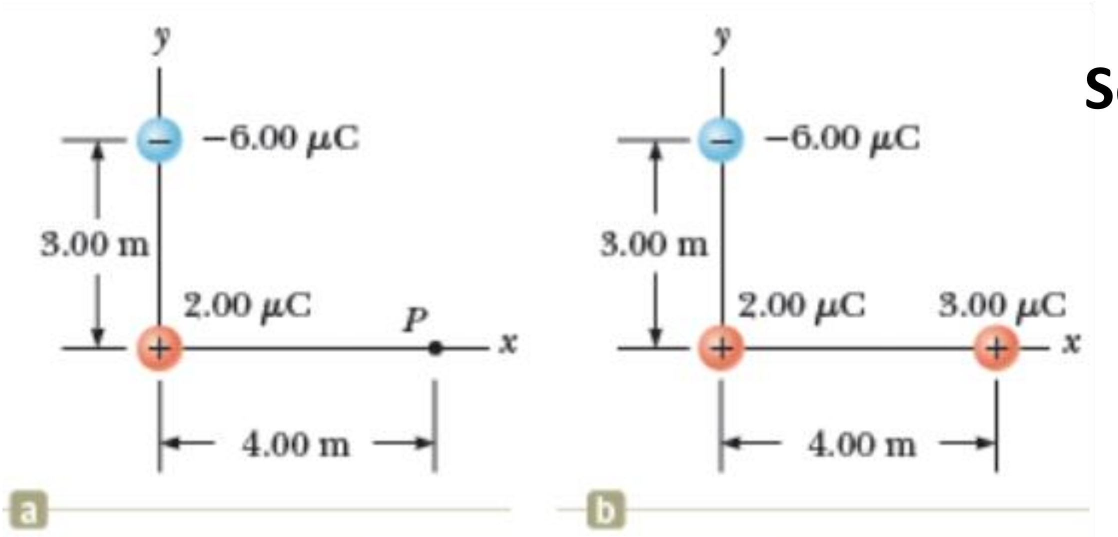
The total potential energy of the system of three charges

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

Example As shown in Fig.a, a charge $q_1 = 2.00\ \mu$C is located at the origin and a

2

to these charges at the point *P*, whose coordinates are (4.00, 0) m. **(B)** Find the change in potential energy of the system of two charges plus a third charge $q_3 = 3.00\ \mu C$ as



Solution: (b)

$$U_f = q_3 V_P$$

$$\Delta U = U_f - U_i = q_3 V_P - 0$$

$$= (3.00 \times 10^{-6}\ \text{C})(-6.29 \times 10^3\ \text{V})$$

$$= -1.89 \times 10^{-2}\ \text{J}$$

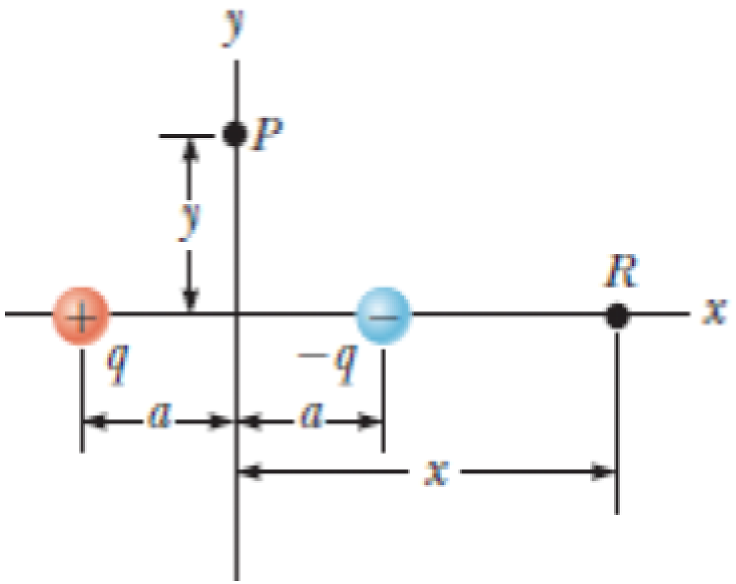
Solution: (a) $V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$

$$V_P = (8.988 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{2.00 \times 10^{-6}\ \text{C}}{4.00\ \text{m}} + \frac{-6.00 \times 10^{-6}\ \text{C}}{5.00\ \text{m}}\right) = -6.29 \times 10^3\ \text{V}$$

Example The Electric Potential Due to a Dipole

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance

is centered at the origin. **(A)** Calculate the electric potential at point *P* on the *y* axis.



$$V_P = k_e \sum_i \frac{q_i}{r_i} = k_e \left(\frac{q}{\sqrt{a^2 + y^2}} + \frac{-q}{\sqrt{a^2 + y^2}} \right) = 0$$

(B) Calculate the electric potential at point *R* on the positive *x* axis.

$$V_R = k_e \sum_i \frac{q_i}{r_i} = k_e \left(\frac{-q}{x - a} + \frac{q}{x + a} \right) = -\frac{2k_e qa}{x^2 - a^2}$$

(C) Calculate *V* and E_x at a point on the *x* axis far from the dipole.

$$V_R = \lim_{x >> a} \left(-\frac{2k_e qa}{x^2 - a^2} \right) \approx -\frac{2k_e qa}{x^2} \quad (x >> a) \qquad E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_e qa}{x^2} \right)$$

Definition of Capacitor and Capacitance

Consider two conductors as shown in Fig. Such a

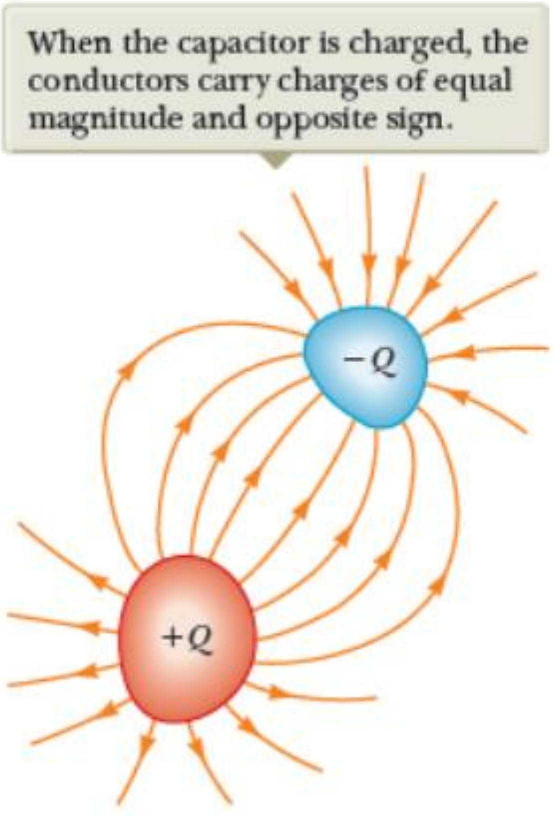
conductors are called *plates.* If the conductors carry charges equal magnitude and opposite sign, a potential

Experiments show that the quantity of charge Q on a capacitor is linearly proportional to the potential difference

$$Q \propto \Delta V$$

$$Q = C\Delta V \qquad \therefore C = \frac{Q}{\Delta V}$$

Where, the proportionality constant depends on the shape and separation of the conductors and *is* **capacitance.** The SI



A capacitor consists of two conductors.

Calculating Capacitance

The calculation is relatively easy if the geometry of the capacitor is simple.

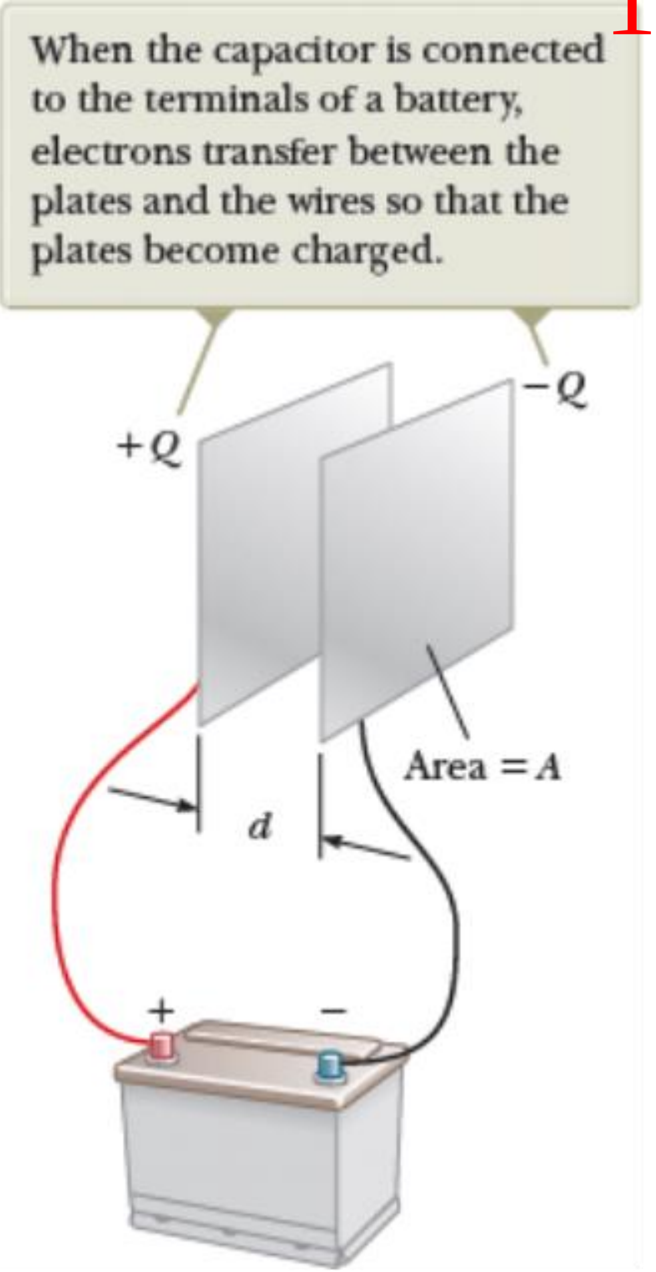
Parallel-Plate Capacitors

Two parallel, metallic plates of equal area *A* are separated by a distance *d* as shown in Fig. One plate

The value of the electric field between the plates is $E = \frac{Q}{\epsilon_0 A}$. The magnitude of the potential difference between the plates = *Ed.*

$$\therefore \Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A} = \frac{\epsilon_0 A}{d}$$



Combinations of Capacitors: Parallel Combination

From Fig.

1 2

$$Q_{total} = Q_1 + Q_2$$

1 1 2 2

The equivalent capacitor

$$Q_{total} = C_{eq}\Delta V$$

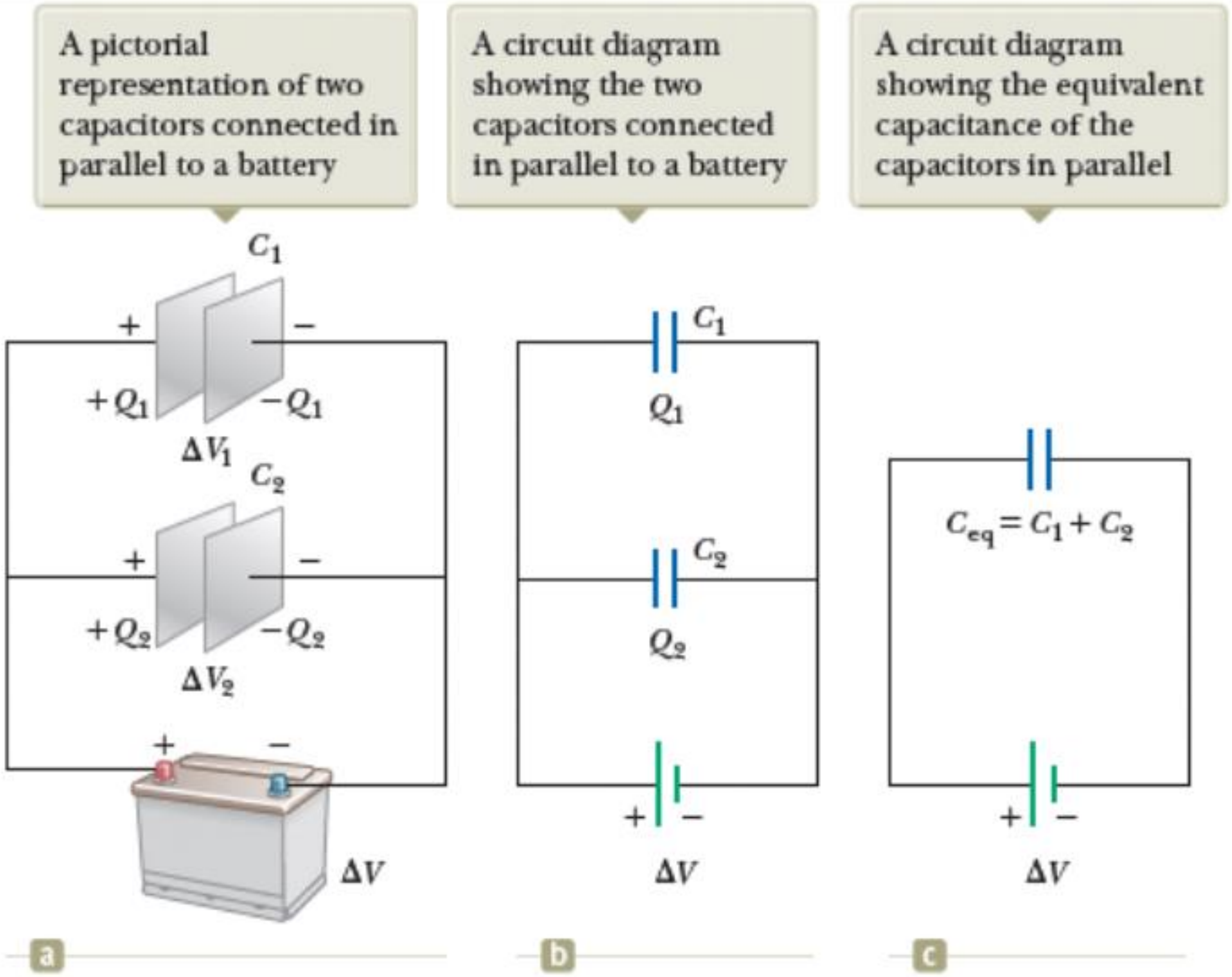
eq 1 1 2 2

$$\Rightarrow C_{eq}\Delta V = C_1 \Delta V + C_2 \Delta V$$

eq 1 2

For parallel:

eq 1 2 3



Combinations of Capacitors: Series Combination

Series combination of

Fig.

1 2

We know

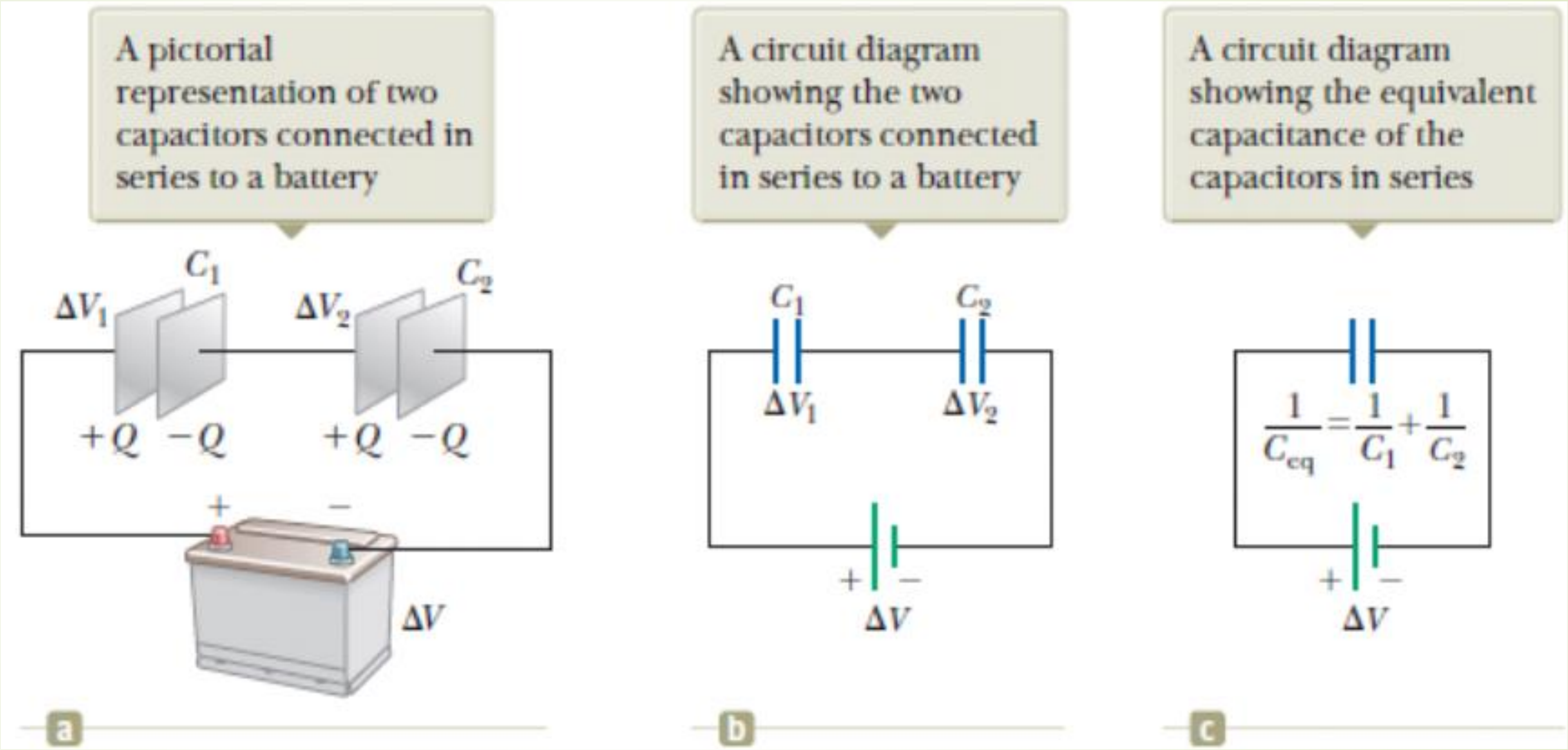
Tot

$= \Delta V_1 + \Delta V_2$

$= \dfrac{Q_1}{C_1} + \dfrac{Q_2}{C_2}$

We know

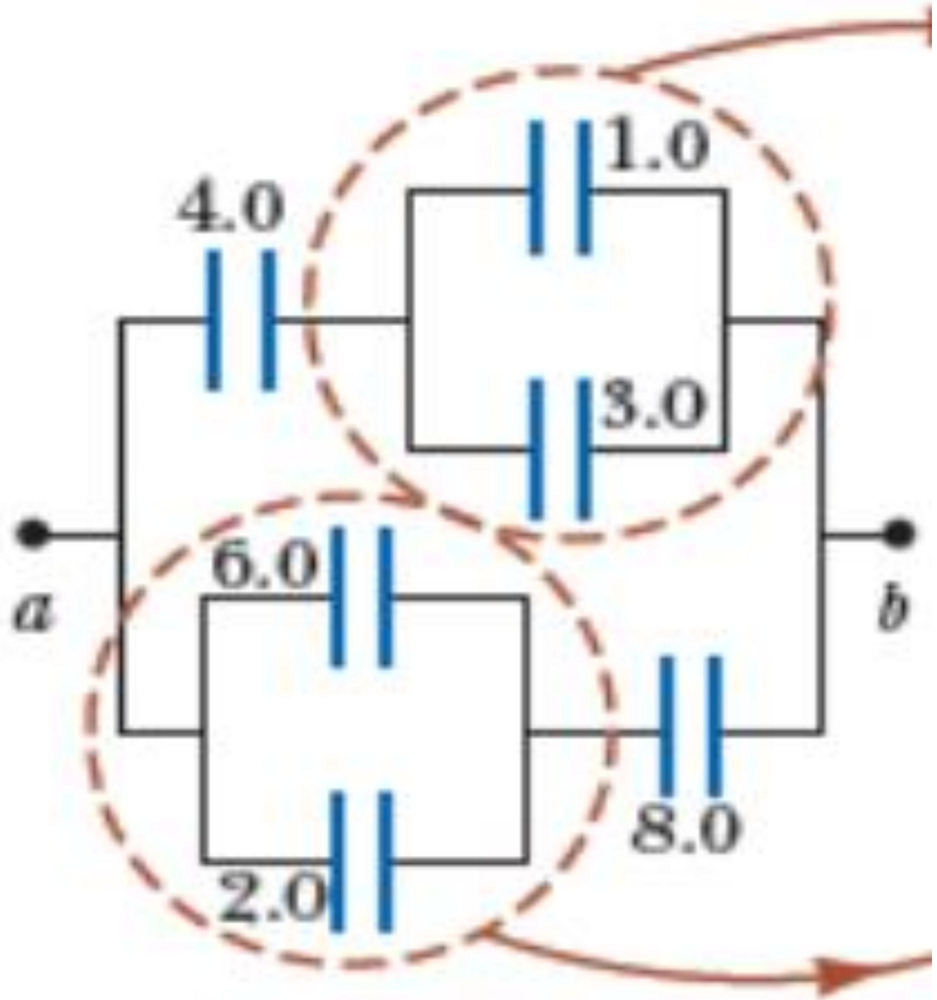
$\Delta V_{Tot} = \dfrac{Q}{C_{eq}}$



$$\therefore \frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \quad \Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

Problem-25

Find the equivalent capacitance between *a* and *b* for the combination capacitances are in microfarads.



$$C_{ea} = C_1 + C_2 = 4.0\ \mu F$$

$$C_{eq} = C_1 + C_2 = 8.0\ \mu F$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu F} + \frac{1}{4.0\ \mu F} = \frac{1}{2.0\ \mu F} \qquad C_{eq} = 2.0\ \mu F$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{8.0\ \mu F} + \frac{1}{8.0\ \mu F} = \frac{1}{4.0\ \mu F} \qquad C_{eq} = 4.0\ \mu F$$

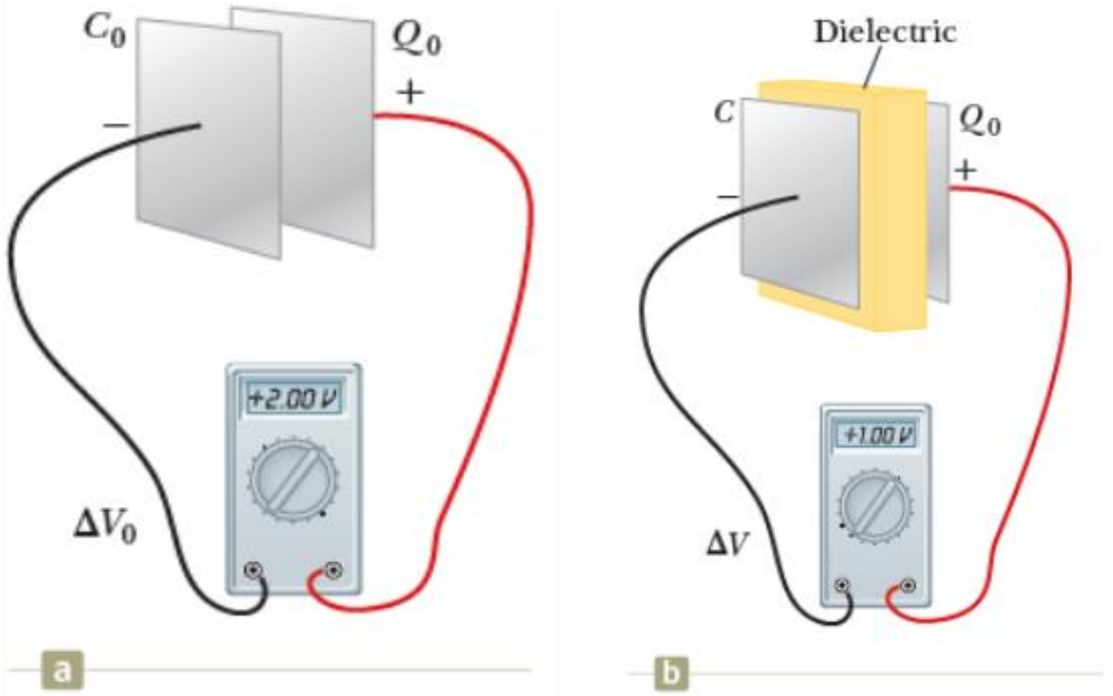
$$C_{eq} = C_1 + C_2 = \boxed{6.0\ \mu F}$$

Capacitors with Dielectrics

A **dielectric** is a nonconducting material such as rubber, glass, or

capacitor that without a dielectric has
$_0$ and a capacitance C_0 . The

capacitor is $\Delta V_0 = \frac{Q_0}{c_0}$ or $C_0 = \frac{Q_0}{V_0}$.



If a dielectric is now inserted between the plates as in Fig.b, the voltmeter indicates that the voltage between the plates decreases to a value ΔV. The voltages with and

0

dielectric constant $k > 1$. The new capacitance:

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0 / k} = k \frac{Q_0}{\Delta V_0} = kC_0 = k \frac{\epsilon_0 A}{d}$$

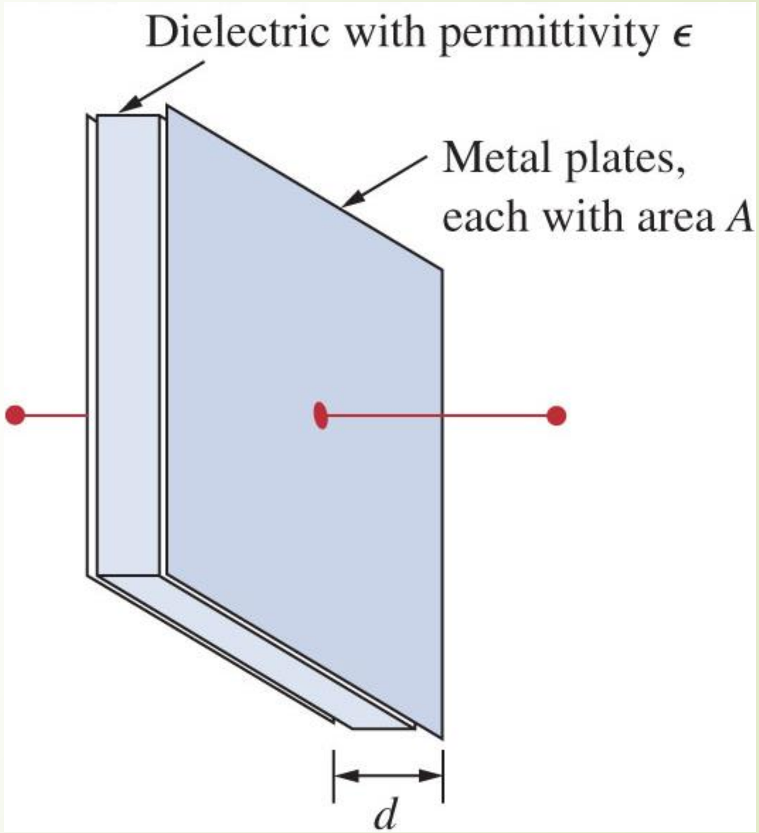
[as $C_0 = \frac{\epsilon_0 A}{d}$]

Table 26.1 Approximate Dielectric Constants

Material	Dielectric Constant κ
Air (dry)	1.000 59
Bakelite	4.9
Fused quartz	3.78
Mylar	3.2
Neoprene rubber	6.7
Nylon	3.4
Paper	3.7
Paraffin-impregnated paper	3.5
Polystyrene	2.56
Polyvinyl chloride	3.4
Porcelain	6
Pyrex glass	5.6
Silicone oil	2.5
Strontium titanate	233
Teflon	2.1
Vacuum	1.000 00
Water	80

Capacitors

- A capacitor is a passive element that stores energy in its electric field
- It consists of two conducting plates separated by an insulator (or dielectric)
- The plates are typically aluminum foil
- The dielectric is often air, ceramic, paper, plastic, or mica



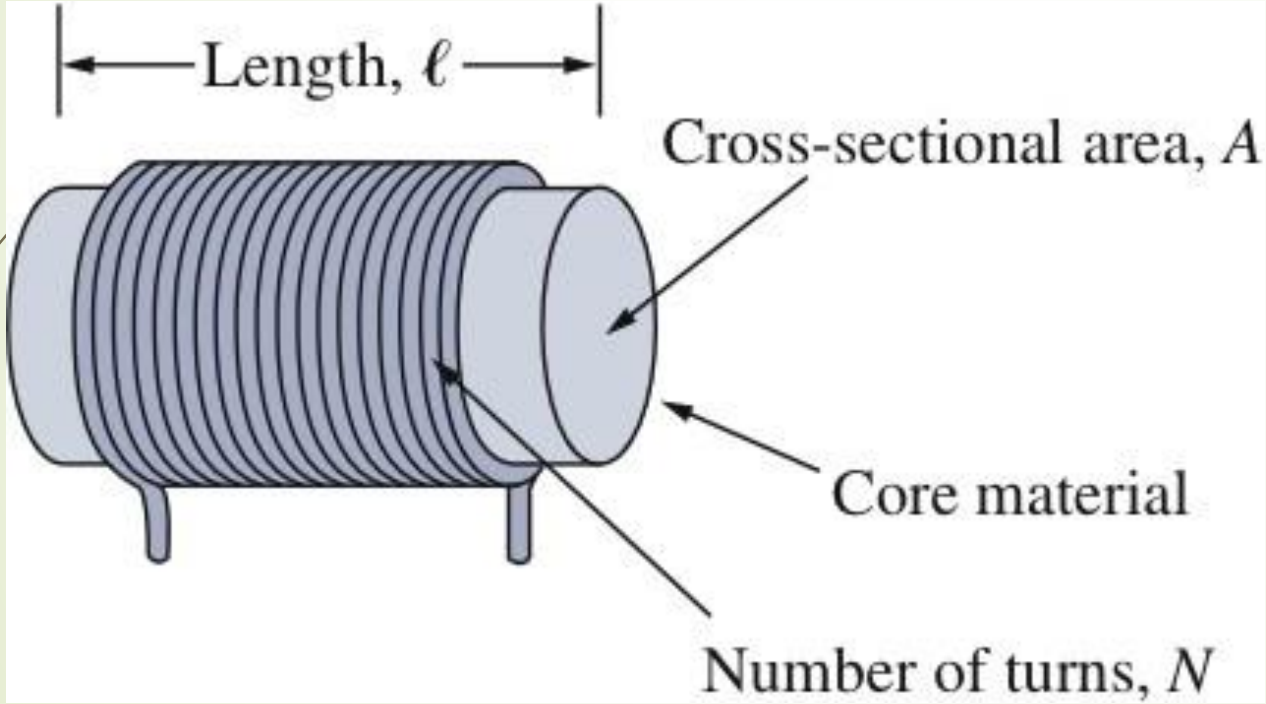
- When a voltage source v is connected to the capacitor, the source deposits a positive charge q on one plate and a negative charge $-q$ on the other.
- The charges will be equal in magnitude
- The amount of charge is proportional to the voltage:

$$q = Cv$$

- Where C is the capacitance

Inductors

An inductor is made of a coil of conducting wire

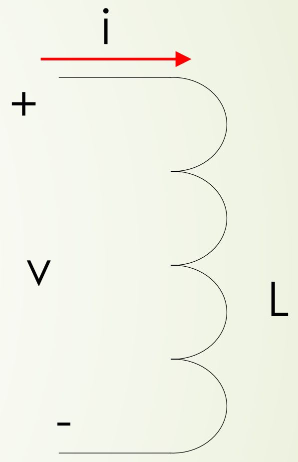


Inductors

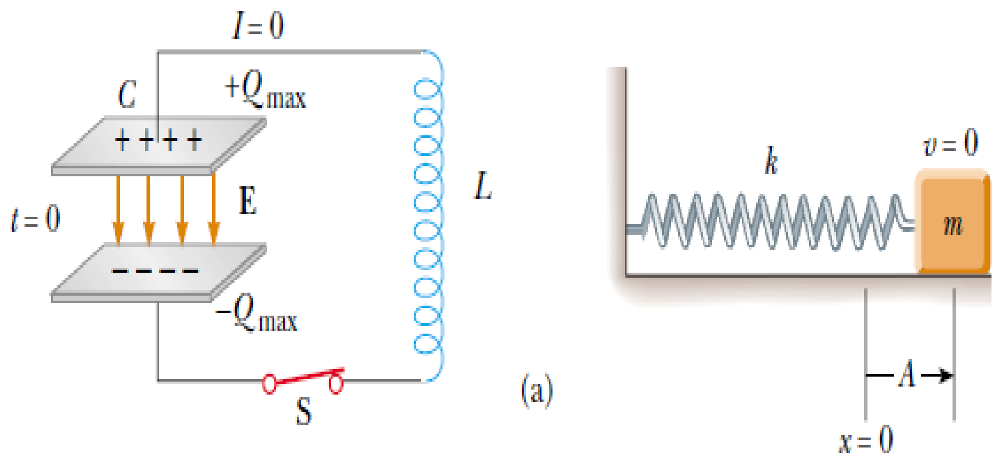
The relation between the flux in inductor and the current through the inductor is given below.

$$\varphi = Li$$

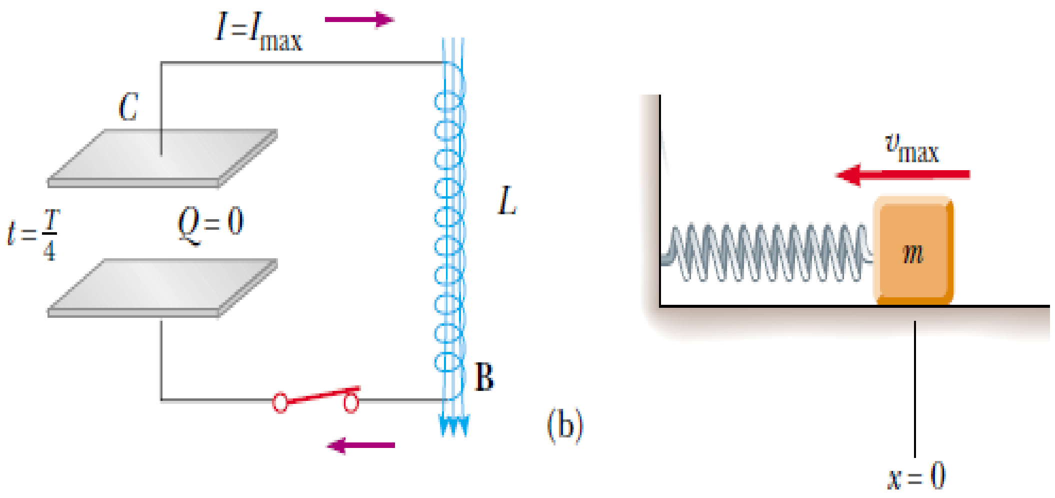
$$v = -\frac{d\varphi}{dt} = -L\frac{di}{dt}$$



An inductor is a passive element designed to store energy in the magnetic field while a capacitor stores energy in the electric field.



I = 0
C
+Q max
+ + + +
E
t = 0
− − − −
-Q max
L
S
(a)
k
v = 0
m
A
x = 0



I = I max
C
L
t = T/4
Q = 0
B
(b)
v max
m
x = 0

LC Circuit

LC circuit has a combination of a pure inductor with zero resistance and a pure capacitor with infinite resistance

As usual we start with an idealised situation where we assume that the resistance in the circuit is negligible. This is analogous to the assumption for mechanical systems that there are no frictional forces present. Initially, the switch is open and the capacitor is charged to voltage V_C. The charge q on the capacitor is given by $q = V_C C$ where C is the capacitance. When the switch is closed the charge begins to flow through the inductor and a current $I = \mathrm{d}q/\mathrm{d}t$ flows in the circuit. This is a time-varying current and produces a voltage across the inductor given by $V_L = L\mathrm{d}I/\mathrm{d}t$. We can analyse the LC circuit using *Kirchhoff's law*, which states that 'the sum of the voltages around the circuit is zero', i.e. $V_C + V_L = 0$. Therefore

$$\frac{q}{c} + L\frac{di}{dt} = 0$$

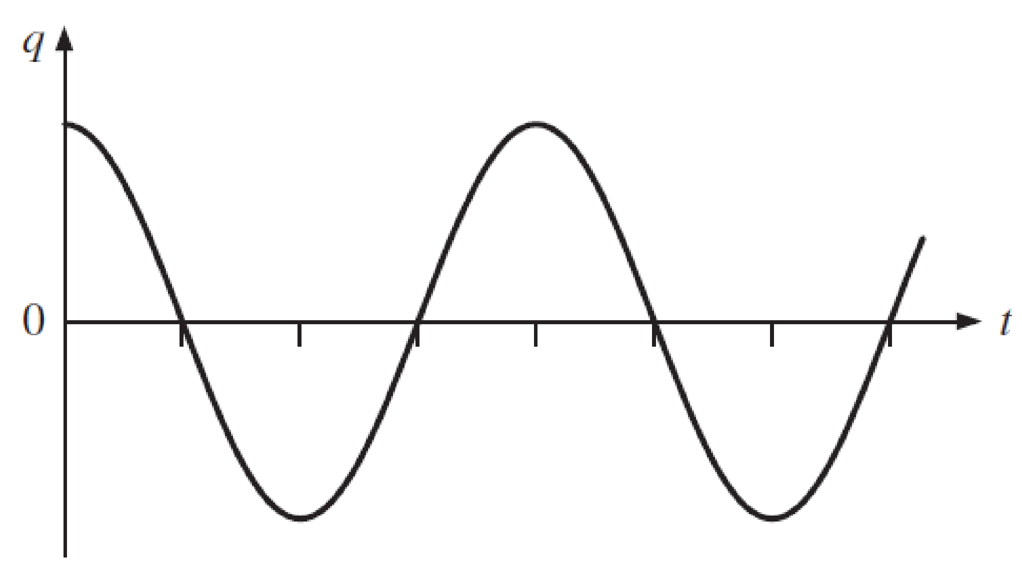
$$\Longrightarrow \frac{q}{c} + L\frac{d^2q}{dt^2} = 0$$

$$\Longrightarrow \frac{d^2q}{dt^2} = -\frac{1}{LC}q = -\omega^2 q$$

$$\Longrightarrow \frac{d^2q}{dt^2} + \omega^2 q = 0$$

LC Circuit

This equation describes how the charge on a plate of the capacitor varies with time. It is of the same form as Equation (1.6) and represents SHM. The frequency of the oscillation is given directly by, $\omega = \sqrt{1/LC}$. Since we have the initial condition that the charge on the capacitor has its maximum value at $t = 0$, then the solution to Equation (1.54) is $q = q_0 \cos \omega t$, where q_0 is the initial charge on the capacitor. The variation of charge q with respect to t is shown in Figure 1.22 and is analogous to the way the displacement of a mass on a spring varies with time.



LC Circuit

We can also consider the energy of this electrical oscillator. The energy stored in a capacitor charged to voltage V_C is equal to $\frac{1}{2}CV_C^2$. This is electrostatic energy. The energy stored in an inductor is equal to $\frac{1}{2}LI^2$ and this is magnetic energy. Thus the total energy in the circuit is given by

$$E = \frac{1}{2}LI^2 + \frac{1}{2}CV_C^2$$

$$E = \frac{1}{2}LI^2 + \frac{1}{2}\frac{q^2}{C}$$

For these electrical oscillations the charge flows between the plates of the capacitor and through the inductor, so that there is a continuous exchange between electrostatic and magnetic energy.

10th Week

Topic: Electricity & Magnetism:

Dielectrics and piezoelectricity, Kirchhoff's Current Law (KCL), Kirchhoff's Voltage Law (KVL), Topic Related problems,

Topic Related Math

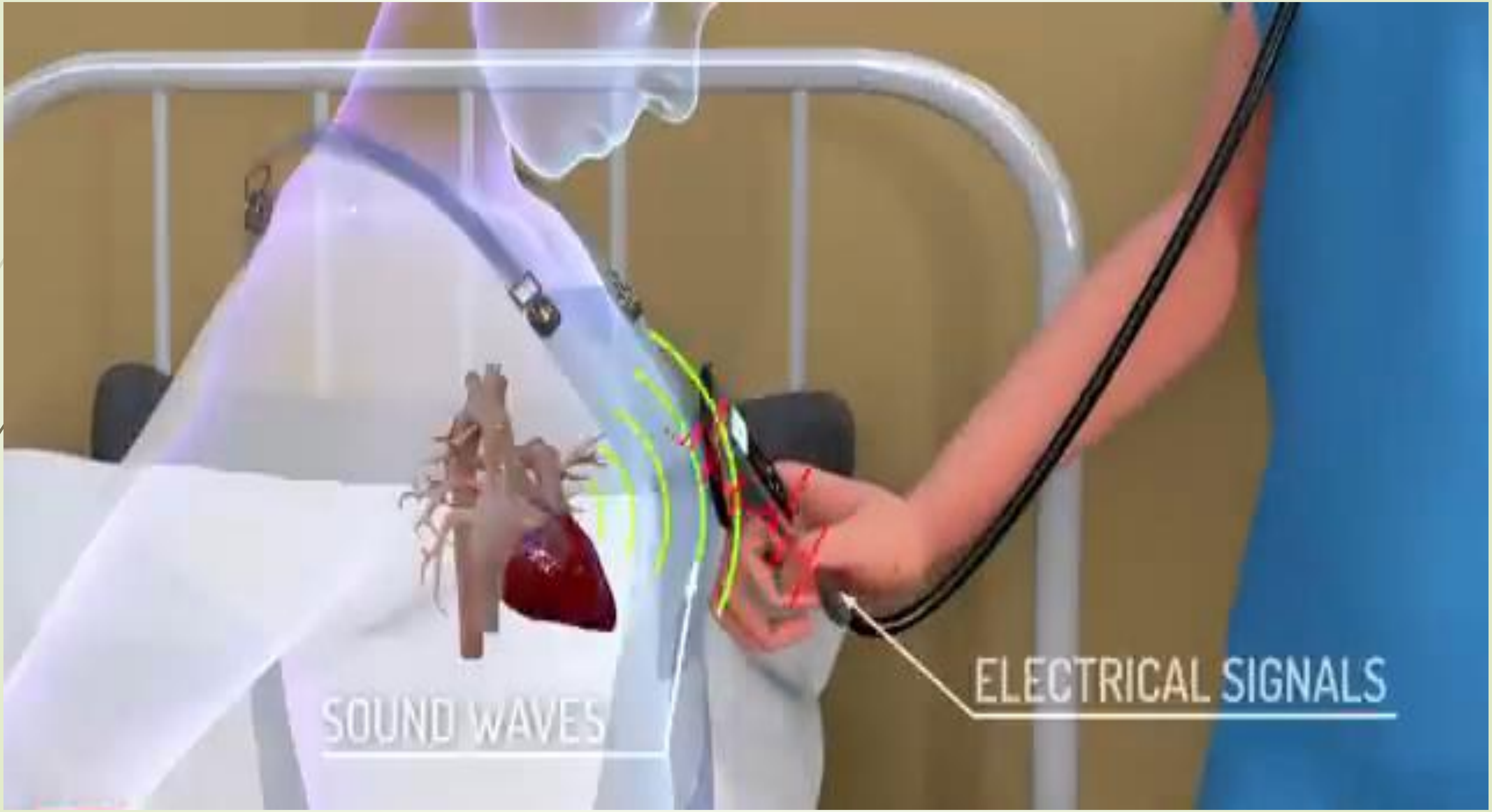
Page: 187- 195

Piezoelectricity

- **Piezoelectricity** is the ability of certain materials to generate an electrical charge in response to applied mechanical stress. This effect occurs because the material's internal structure lacks a center of symmetry, allowing mechanical deformation to create an electric polarization .

- **Direct Piezoelectric Effect**: When mechanical stress (compression, tension, or vibration) is applied to a piezoelectric material, it generates an electric charge. This means that mechanical energy is directly converted into electrical energy.

- **Reverse Piezoelectric Effect**: When an electric field or voltage is applied to a piezoelectric material, it undergoes a mechanical deformation (expansion or contraction). In this case, electrical energy is directly converted into mechanical energy.


SOUND WAVES
ELECTRICAL SIGNALS

Dielectric materials:

Dielectric materials are insulating substances that do not conduct electricity but can support electrostatic fields. They are characterized by their ability to be polarized in the presence of an electric field, meaning their positive and negative charges shift slightly, aligning with the field. This property makes them essential in various electrical and electronic applications.

Key Properties of Dielectric Materials:

1.Permittivity (ε_o):

1. The measure of a material's ability to permit electric field lines. Higher permittivity means better dielectric properties.
2. Relative permittivity (ε_r), or the dielectric constant, is the ratio of the material's permittivity to that of free space ($\varepsilon_r \backslash \varepsilon_o$).

2.Polarizability:m

1. The degree to which the material can be polarized. This determines how effectively the material reduces the overall electric field.

3.Dielectric Strength:

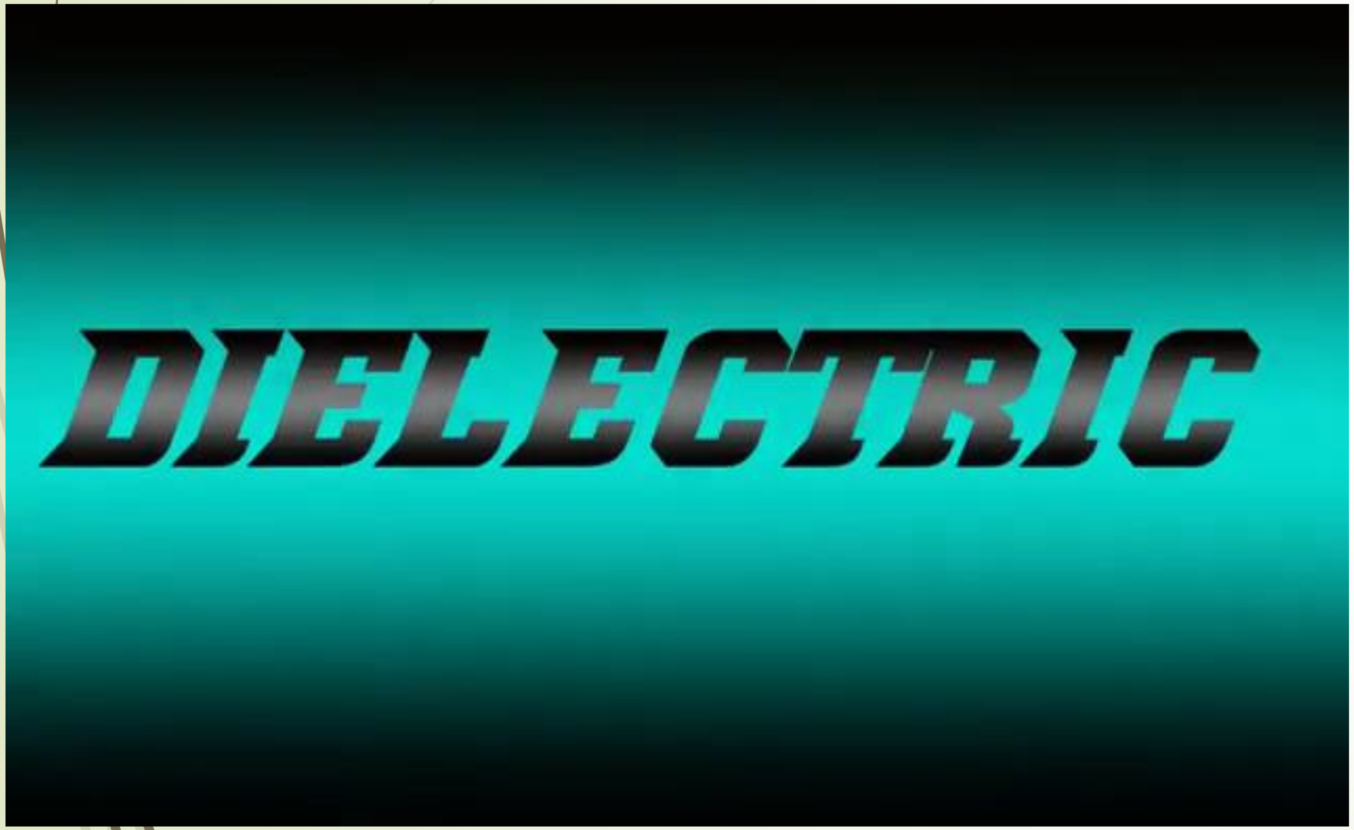
1. The maximum electric field a dielectric material can withstand without breaking down (i.e., becoming conductive).

4.Loss Tangent (tanδ):

1. A measure of the energy lost as heat within the dielectric when subjected to an alternating electric field.

Types of Dielectric Materials:

1. **Polar Dielectrics**:
   1. Molecules have permanent electric dipole moments (e.g., water, certain polymers).
   2. Polarization increases with the applied field and aligns in its direction.
2. **Non-Polar Dielectrics**:
   1. Molecules do not have permanent dipoles but can be polarized when an external field is applied (e.g., gases like nitrogen or solid materials like polyethylene).
3. **Solid Dielectrics**:
   1. Examples include ceramics, glass, and polymers, often used in capacitors and insulators.
4. **Liquid Dielectrics**:
   1. Examples include transformer oil, used in high-voltage applications.
5. **Gas Dielectrics**:
   1. Examples include air, SF_6, and nitrogen, used as insulation in high-voltage equipment.



Applications:

1.Capacitors: Dielectrics store electrical energy by enhancing the capacitance.

2.Insulators: Prevent current flow between conductive parts.

3.Electrostatic Applications: Used in sensors and actuators.

4.Transformers: Liquid dielectrics provide insulation and cooling.

Kirchhoff's Voltage Law (KVL):

KVL states that the sum of all voltages in a closed loop of a circuit is equal to zero. This law is based on the principle of energy conservation.

$$\sum_{i=1}^{n} V_i = 0$$

Where:

- V_i represents the voltage across each element in the loop.
- n is the number of elements in the loop.

Explanation:

- As a charge moves around a closed loop, the energy gained (from sources like batteries) and the energy lost (due to resistances, for example) must balance.
- Voltages across components can be positive (rise) or negative (drop), depending on the direction of traversal.

Example:

In a loop containing a resistor (R), a voltage source (V_s), and a current (I):

$$V_s - IR = 0$$

Kirchhoff's Current Law (KCL):

KCL states that the sum of currents entering a junction (or node) in an electrical circuit is equal to the sum of currents leaving the junction. This is based on the conservation of charge.

$$\sum_{i=1}^{n} I_i = 0$$

Where:

- I_i represents the current in each branch connected to the node.

Explanation:

- The total charge entering a junction must equal the total charge leaving it since there is no charge accumulation at the node.

Example:

If three currents I_1, I_2, and I_3 meet at a node:

$$I_1 + I_2 - I_3 = 0$$

or

$$I_3 = I_1 + I_2$$

Applications of KVL and KCL:

1. **Circuit Analysis**: Both laws are used in mesh analysis (KVL) and nodal analysis (KCL) to solve for unknown currents and voltages.

2. **Electrical Networks**: Fundamental to understanding series and parallel circuits.

3. **Design**: Used in designing and analyzing complex electrical systems like power grids, signal processing circuits, and electronic devices.

11TH WEEK

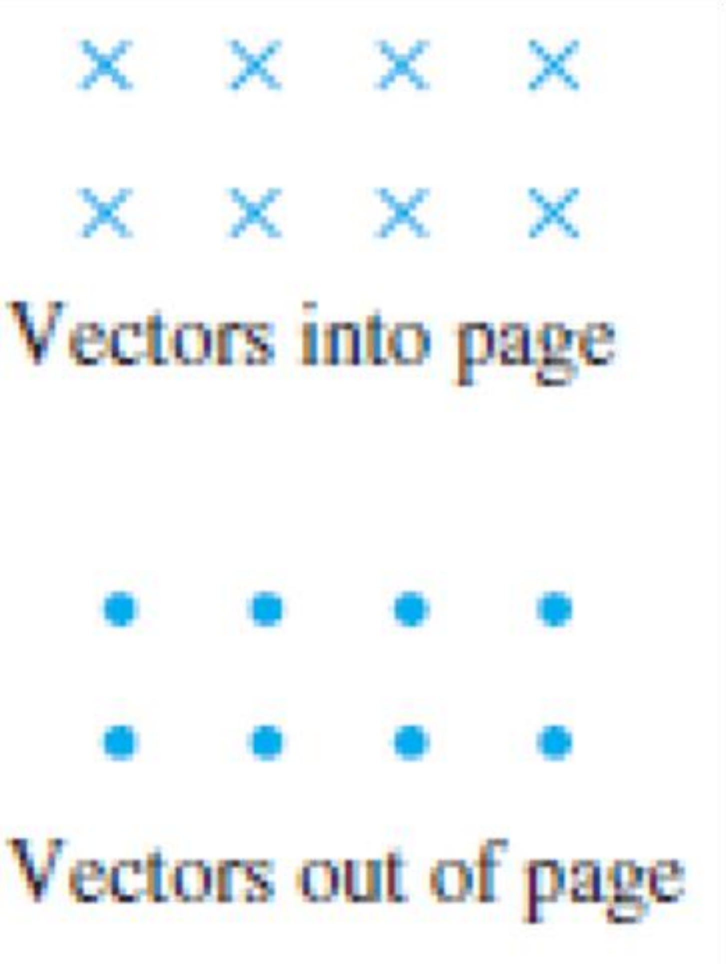
TOPIC: ELECTRICITY & MAGNETISM:

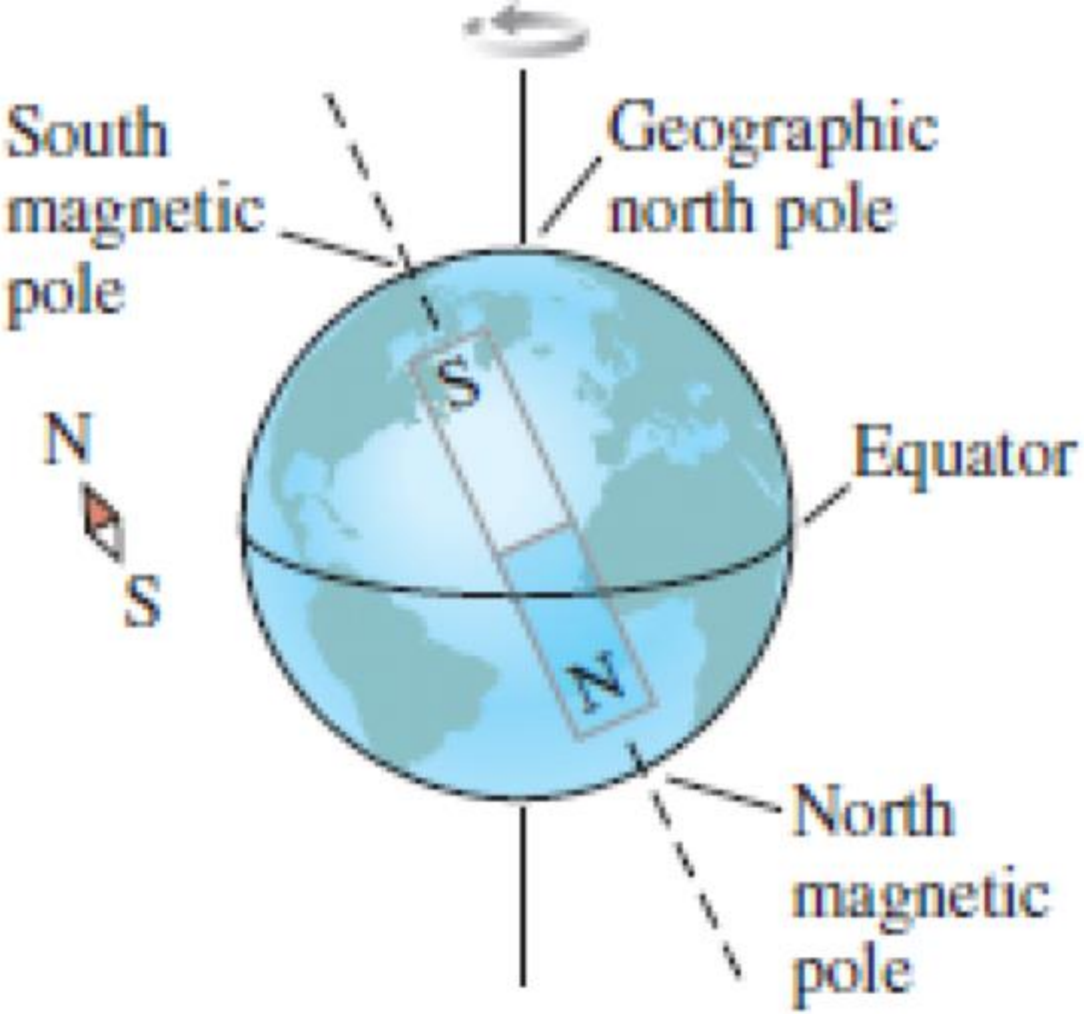
MAGNETISM, MAGNETIC FORCE ON A CONDUCTOR, MAGNETISM RELATED LAW & PROBLEMS

TOPIC RELATED MATH

PAGE: 196-206

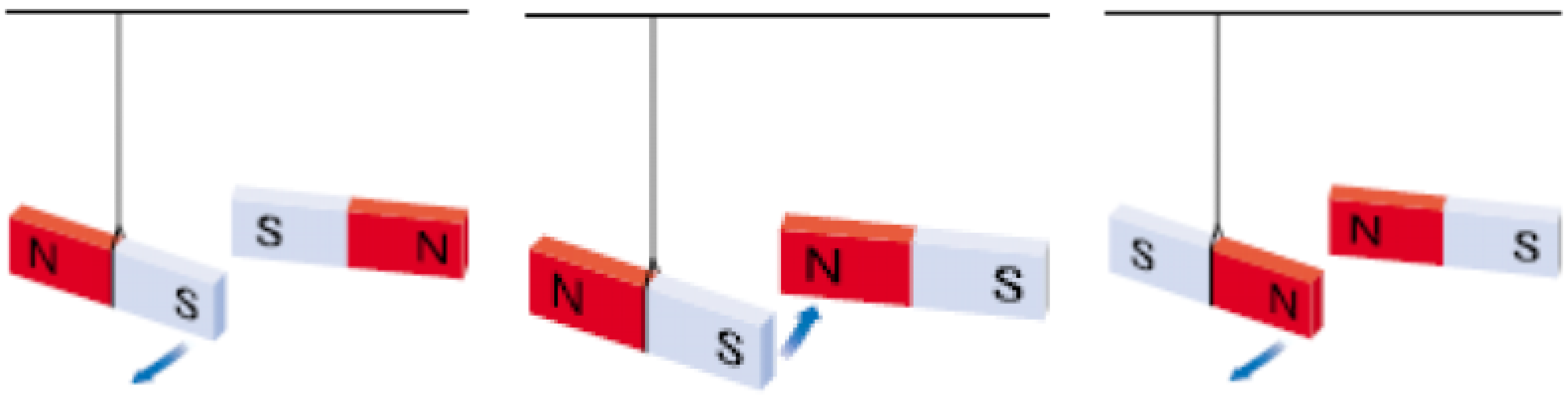
Magnet :The earth is a large magnet.

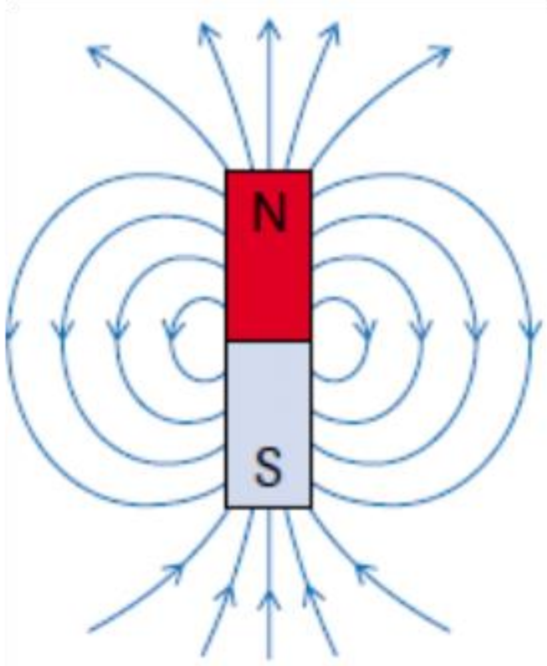




Magnets & Magnetic Fields

Law of Magnetic Poles: Opposite magnetic poles attract. Similar magnetic poles repel.





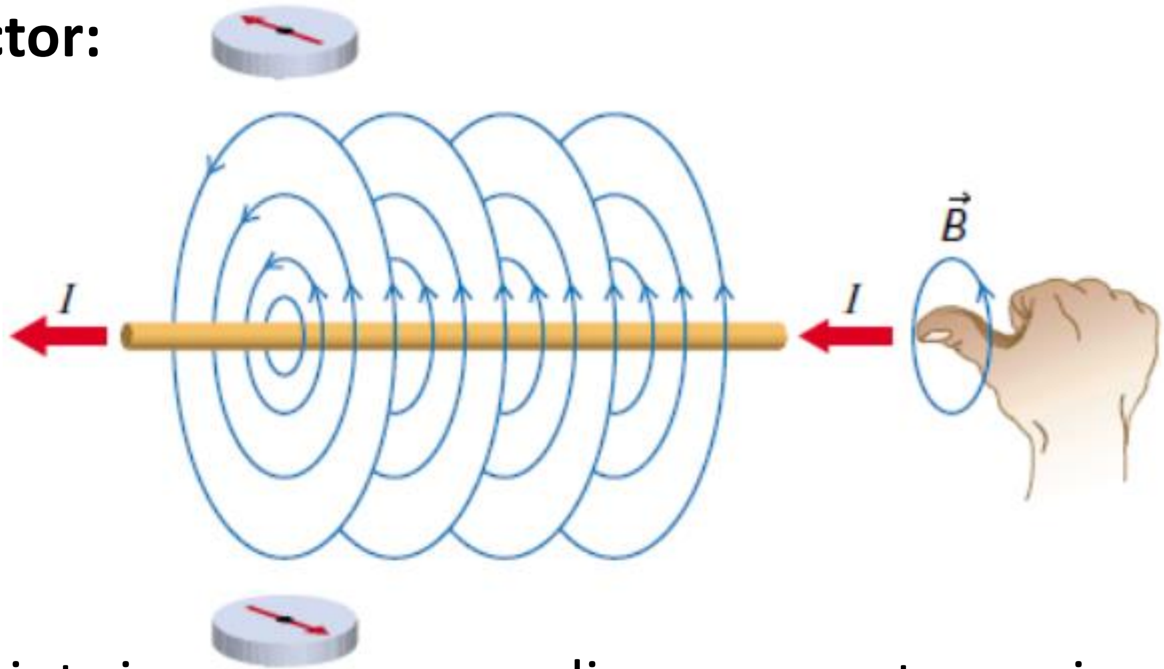
Magnetic force field the area around a magnet in which magnetic forces are exerted

Principle of Electromagnetism: Moving electric charges produce a magnetic field.

Right-Hand Rule for a Straight Conductor:

If a conductor is grasped in the right hand, with the thumb pointing in the

fingers point in the direction of the magnetic field lines.



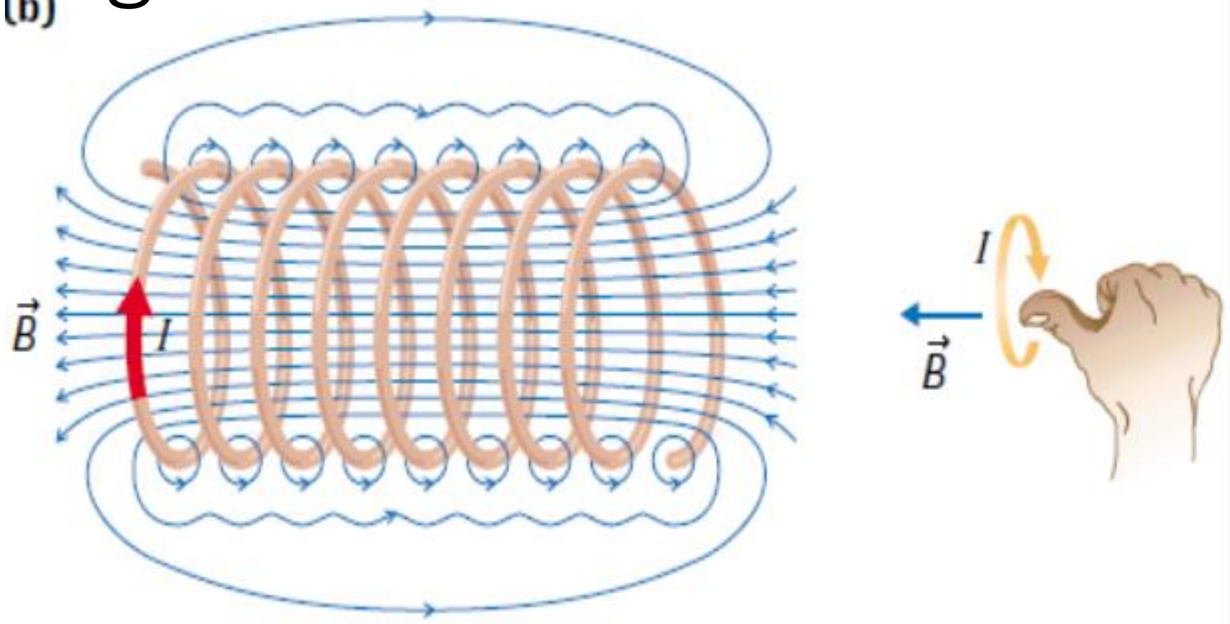
Let us define the **magnetic field** $\vec{B}$ as having the following properties:

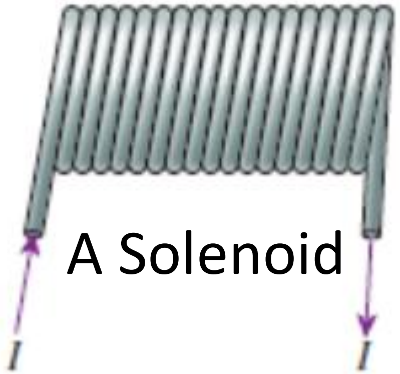
1. A magnetic field is created at *all* points in space surrounding a current-carrying wire.
2.

   call the *magnetic field strength B,* and a direction.
3. The magnetic field exerts forces on magnetic poles. The force on a north pole is parallel to $\vec{B}$; the force on a south pole is opposite $\vec{B}$.

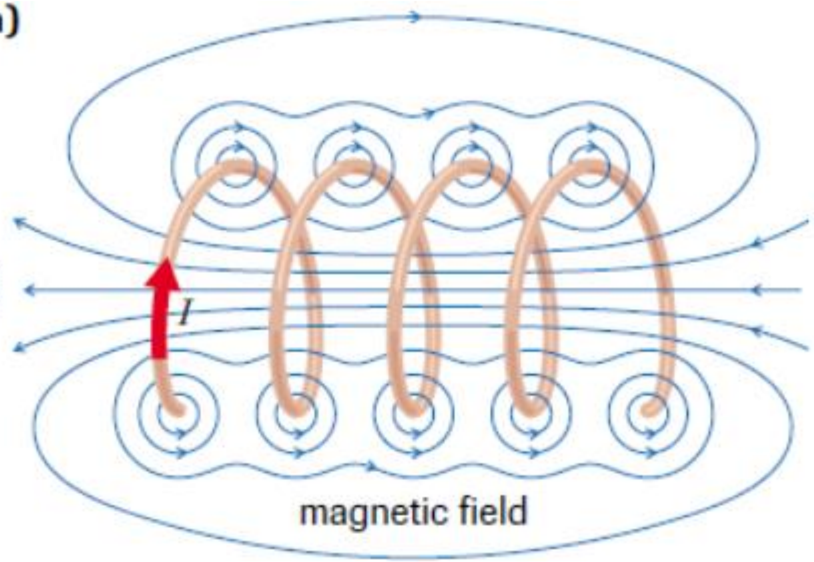
Right-Hand Rule for a Solenoid

(b)





(a)



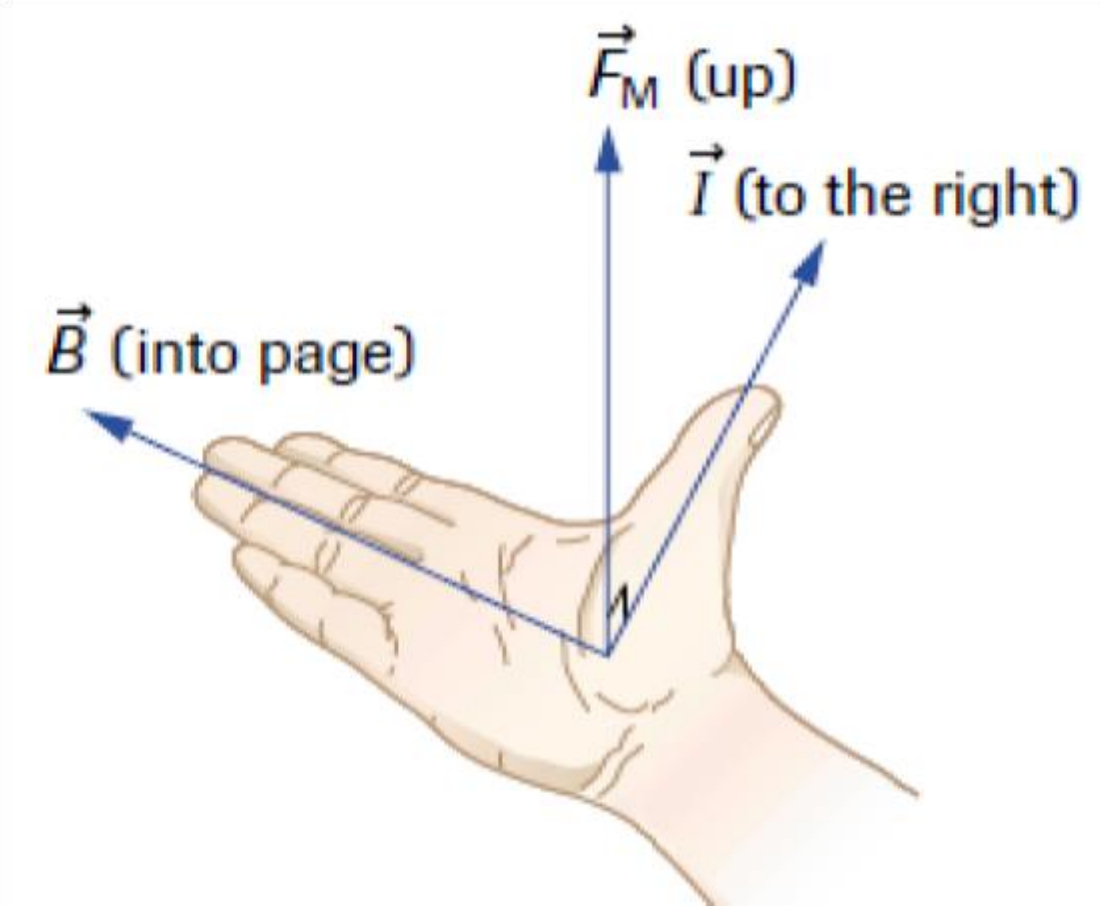
If a solenoid is grasped in the right hand, with the fingers curled in the direction of the electric

magnetic field lines in its core.

Magnetic Force on Moving Charges

The magnitude of the magnetic force F_M on a charged particle

- is directly proportional to the magnitude of the magnetic field $\vec{B}$, the velocity $\vec{v}$, and the charge q of the
- depends on the angle θ between the magnetic field $\vec{B}$ and the velocity $\vec{v}$,



Combining these factors gives

$$F_M = qvB \sin\theta$$

Problem-26

An electron accelerates from rest in a horizontally directed electric field through a potential

electric field, entering a magnetic field of magnitude 0.20 T directed into the page

(a) Calculate the initial speed of the electron upon entering the magnetic field.

(b) Calculate the magnitude and direction of the magnetic force on the electron.

Solution

$\Delta V = 46\,\text{V}$ $\quad v = ?$

$B = 0.20\,\text{T} = 0.20\,\text{kg/C·s}$ $\quad F_M = ?$

$m_e = 9.11 \times 10^{-31}\,\text{kg}$ (from Appendix C)

$q = 1.6 \times 10^{-19}\,\text{C}$

$$-\Delta E_E = \Delta E_K$$

$$q\Delta V = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2q\Delta V}{m}}$$

(b) $\quad F_M = qvB \sin\theta$

Direction?

Assignment

An electron moving through a uniform magnetic field with a velocity of 2.0×10^6 m/s [up] experiences a maximum magnetic force of 5.1×10^{-14} N [left]. Calculate the magnitude and direction of the magnetic field.

Magnetic Force on a Conductor

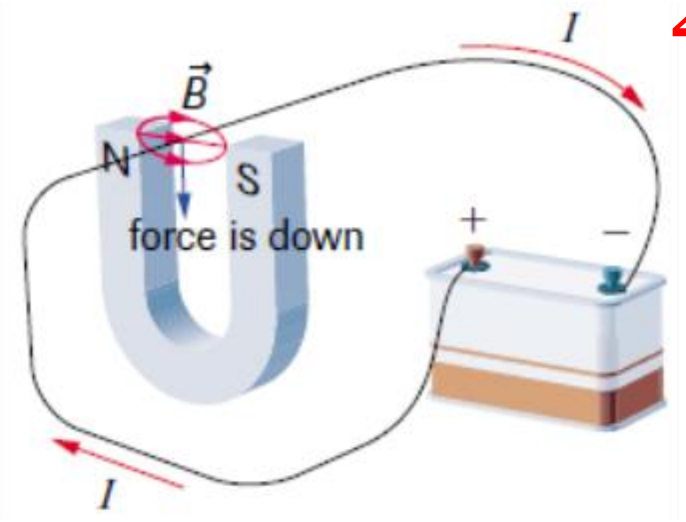
Consider a conductor with a current *I*, placed in a magnetic field of magnitude *B*. The force F on the

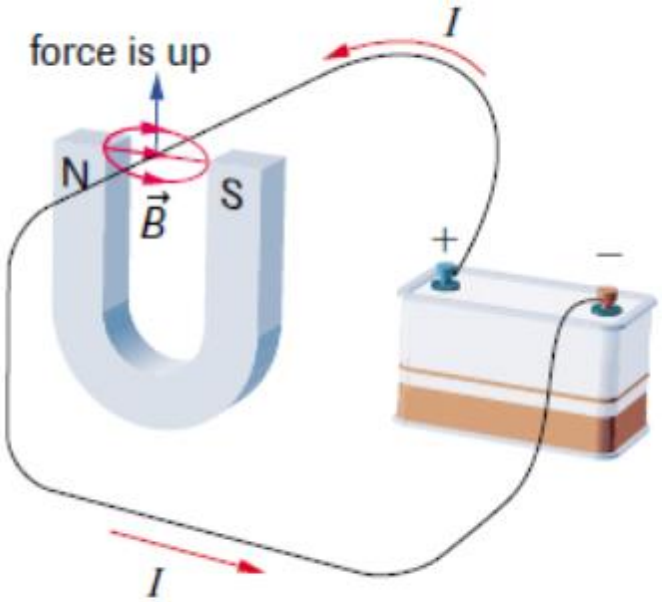
of the magnetic field B, to the current in the conductor I, and to the length of the conductor ℓ.

current) and the magnetic field lines is θ, the of the magnetic force is directly proportional to θ. these

$$F = I\ell B\sin\theta$$

B is the magnitude of the magnetic field strength, in teslas (T).





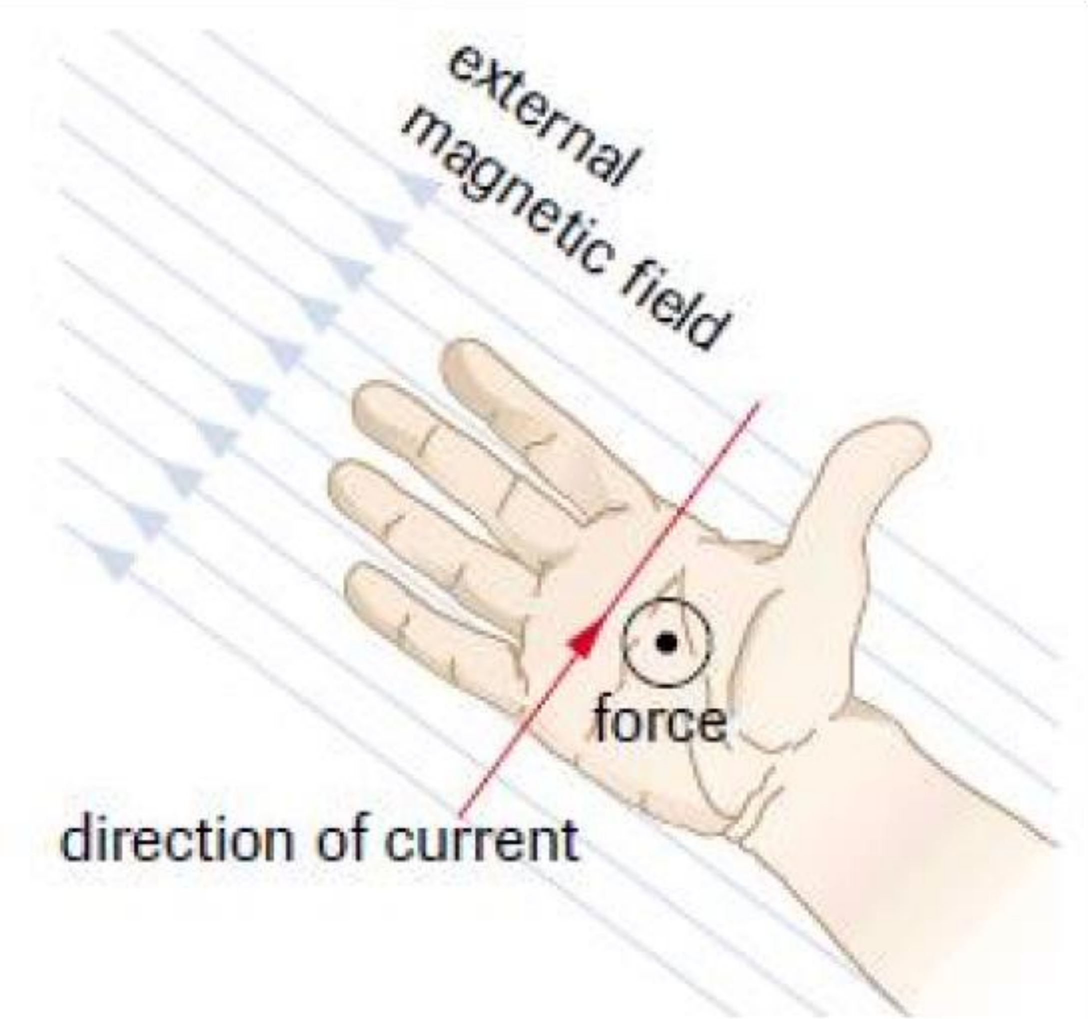
Right-Hand Rule for the Motor Principle

Another simple right-hand rule, equivalent to the one for charges

used to determine the relative $\vec{F}$ I, and B:

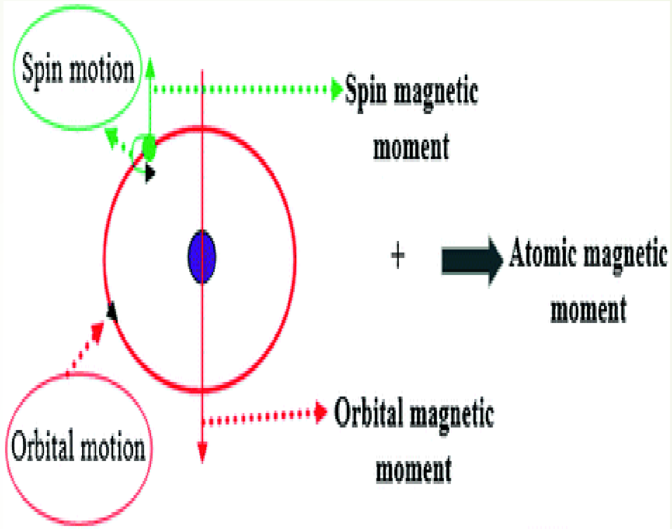
- If the right thumb points in the direction of the current (flow of positive charge), and the extended

  magnetic field, the force is in the direction in which the right palm



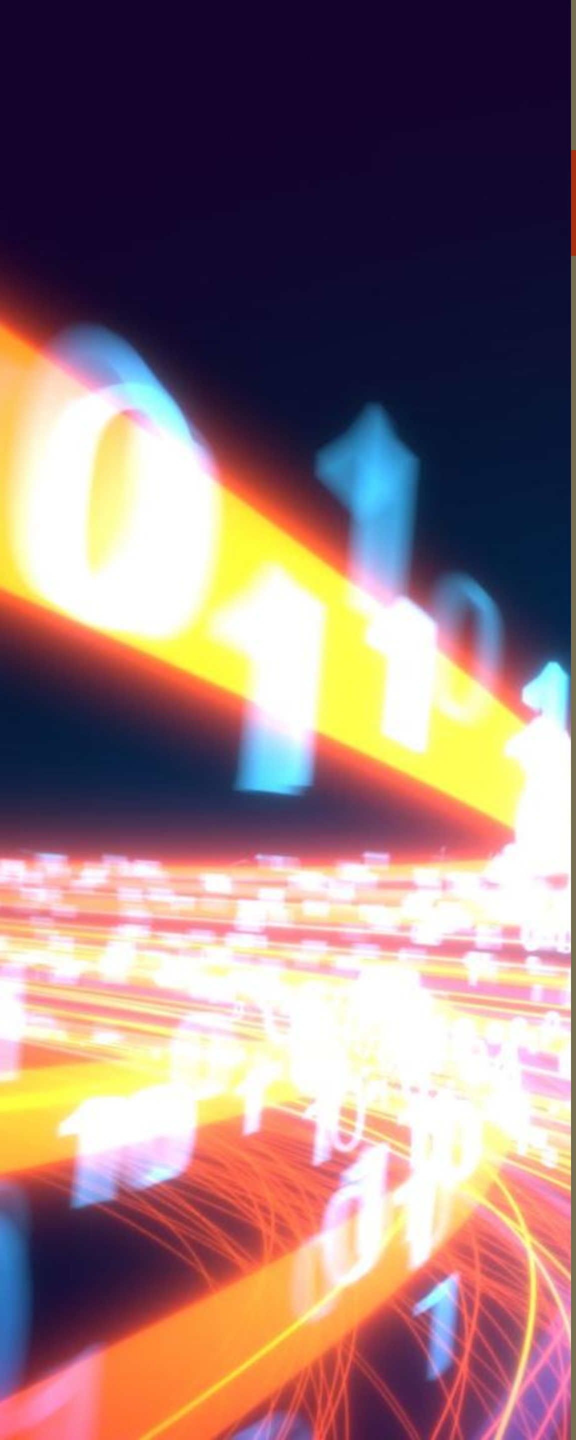
❖ What is Magnetism

Magnetism is the force exerted by magnets when they attract or repel each other. Magnetism is caused by the motion of electric charges.



❖ Origin of magnetism

Magnetism arises from two types of motions of electrons in atoms-one is the motion of the electrons in an orbit around the nucleus, similar to the motion of the planets in our solar system around the sun, and the other is the spin of the electrons around its axis, analogous to the rotation of the Earth about its axis. The orbital and the spin motion independently impart a magnetic moment on each electron causing each of them to behave as a tiny magnet. The magnetic moment of a magnet is defined by the rotational force experienced by it in a magnetic field of unit strength acting perpendicular to its magnetic axis. In a large fraction of the elements, the magnetic moment of the electrons cancels out because of the Pauli Exclusion Principle, which states that each electronic orbit can be occupied by only two electrons of opposite spin.



12th Week

Topic: Electricity & Magnetism:

Hall effect, Type of Magnetism,

Topic Related Problems

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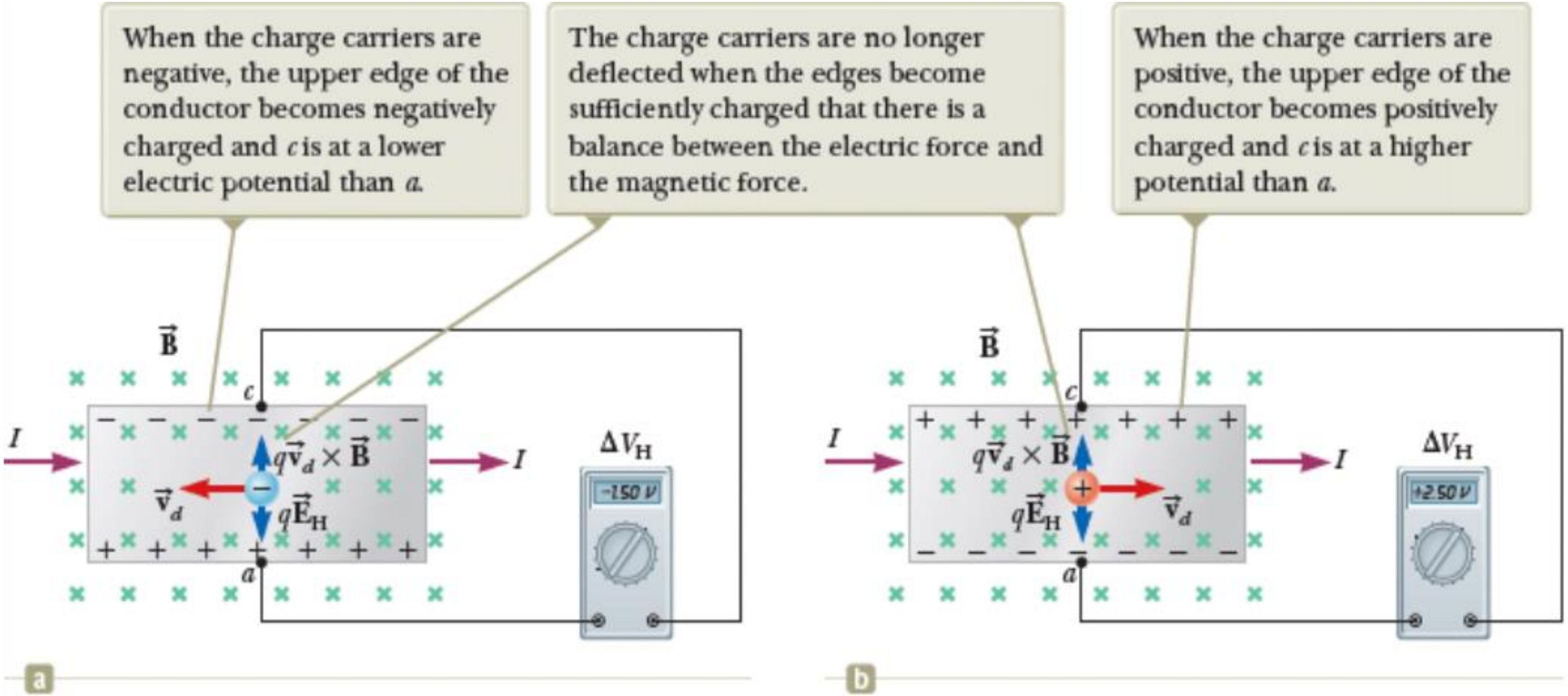
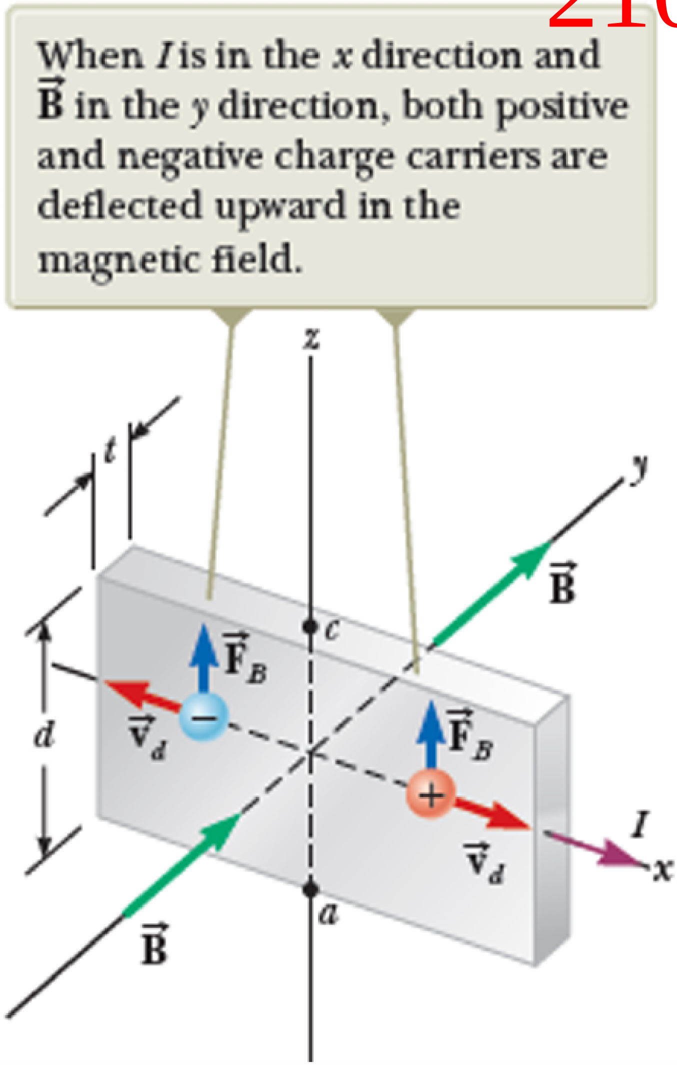
The Hall Effect (Contd.)



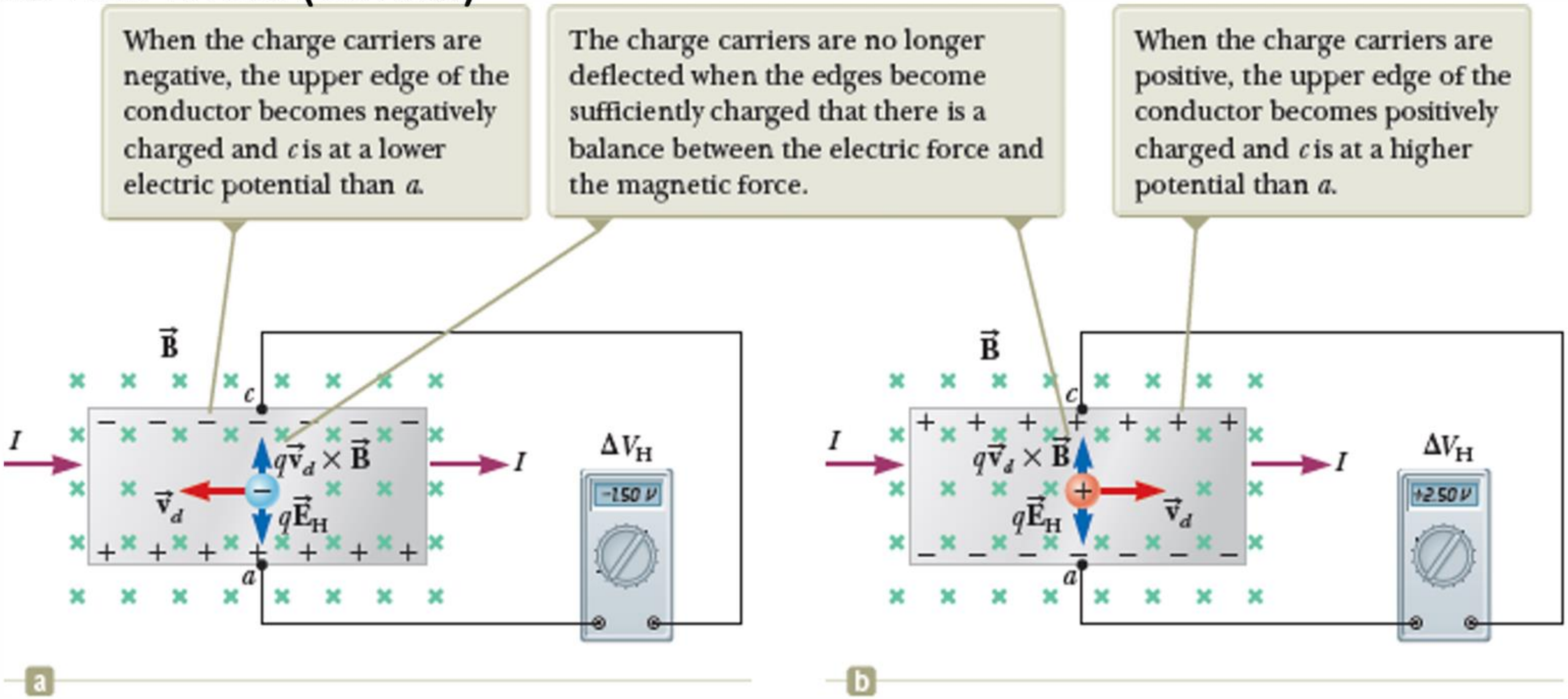
A sensitive voltmeter connected across the sample as shown in Fig. (a & b) can measure the potential difference, known as the **Hall voltage**, generated across the

The Hall Effect

When a current-carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field. This phenomenon is known as the *Hall effect. A* flat conductor carrying a current I in the x direction is shown in Fig. A uniform magnetic field $\vec{B}$ is applied in the y direction. If the charge carriers are electrons moving in the negative x direction with a drift velocity $\vec{v}_d$, they experience an upward magnetic force $\vec{F}_B = q\vec{v}_d \times \vec{B}$, are deflected upward, and accumulate at the upper edge of the flat conductor, leaving an excess of positive charge at the lower edge (Fig. a in next page). This accumulation of charge at the edges establishes an electric field in the conductor and increases until the electric force on carriers remaining in the bulk of the conductor balances the magnetic force acting on the carriers.



The Hall Effect (Contd.)



A sensitive voltmeter connected across the sample as shown in Fig. (a & b) can measure the potential difference, known as the **Hall voltage**, generated across the conductor.

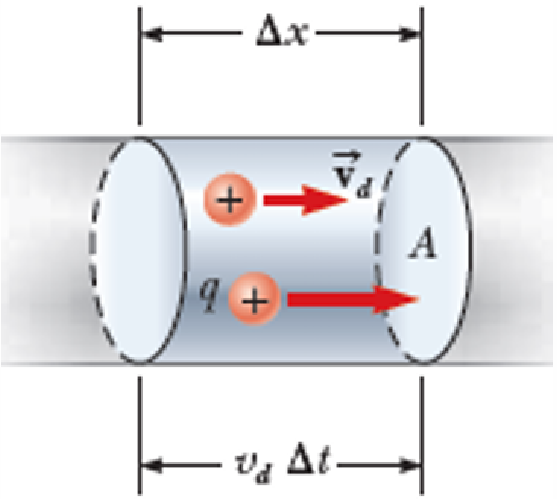
Hall voltage

The magnetic force exerted on the carriers has magnitude qv_dB. In equilibrium, this force is balanced by the electric force qE_H, where E_H is the magnitude of the electric field due to the charge separation (sometimes referred to as the *Hall field*). Therefore,

$$qv_dB = qE_H \qquad \Rightarrow E_H = v_dB$$

If *d* is the width of the conductor, the Hall voltage is

$$\Delta V_H = E_Hd = v_dBd$$



The total charge ΔQ in this segment is, $\Delta Q = (nA\,\Delta x)q = (nAv_d\Delta t)q$, where *n* = the charge carrier density, *q* is the charge on each carrier.

$$\therefore I = \frac{\Delta Q}{\Delta t} = (nAv_d)q \qquad \Rightarrow v_d = I/nAq$$

$$\Delta V_H = \frac{IBd}{nAq} = \frac{IBd}{n(td)q} = \frac{IB}{ntq}$$ Where, *t* is the thickness of the conductor.

Assignment:

The accompanying table shows measurements of the Hall voltage and corresponding magnetic field for a

data and deduce a relationship between the two variables. (b) If the measurements were taken with a

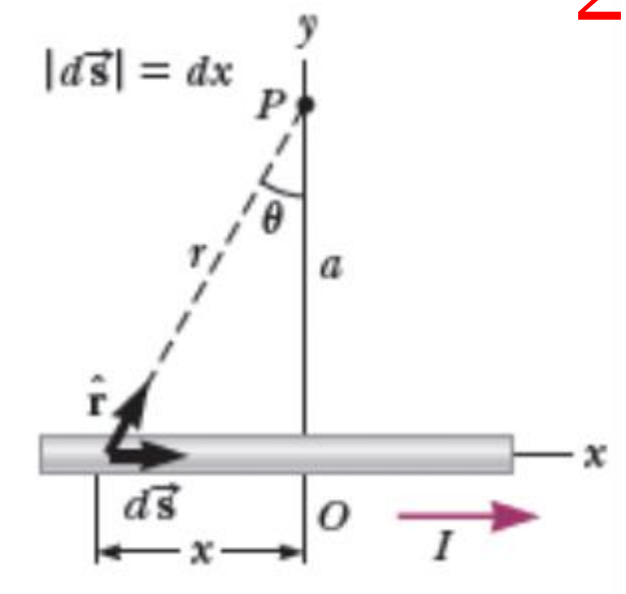
material having a charge-carrier density of $1.00 \times$ ^{26} carriers/m^3, what is the thickness of the sample?

ΔV_H (μV)	B (T)
0	0.00
11	0.10
19	0.20
28	0.30
42	0.40
50	0.50
61	0.60
68	0.70
79	0.80
90	0.90
102	1.00

Example:

Consider a thin, straight wire of finite length carrying a constant current *I* and placed along the *x* axis as shown in

magnetic field at point *P* due to this current.

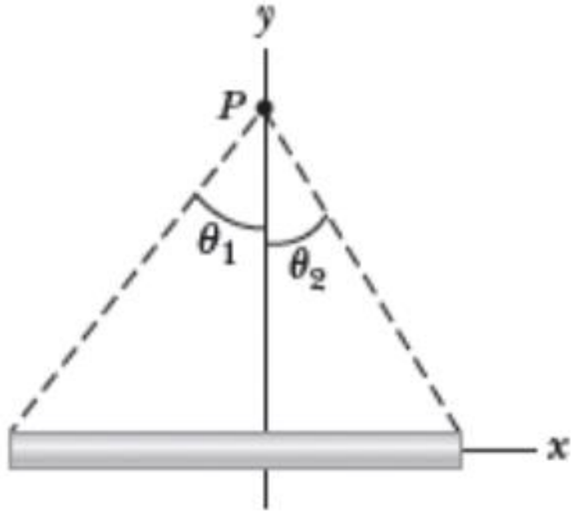


$$d\vec{s} \times \hat{r} = |d\vec{s} \times \hat{r}|\,\hat{k} = \left[dx \sin\left(\frac{\pi}{2} - \theta\right)\right]\hat{k} = (dx\cos\theta)\hat{k}$$

$$(1)\quad d\vec{B} = (dB)\hat{k} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\hat{k} \qquad (2)\quad r = \frac{a}{\cos\theta}$$

$$x = -a\tan\theta \qquad (3)\quad dx = -a\sec^2\theta\, d\theta = -\frac{a\,d\theta}{\cos^2\theta}$$

$$(4)\quad dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a\,d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right)\cos\theta = -\frac{\mu_0 I}{4\pi a}\cos\theta\, d\theta$$



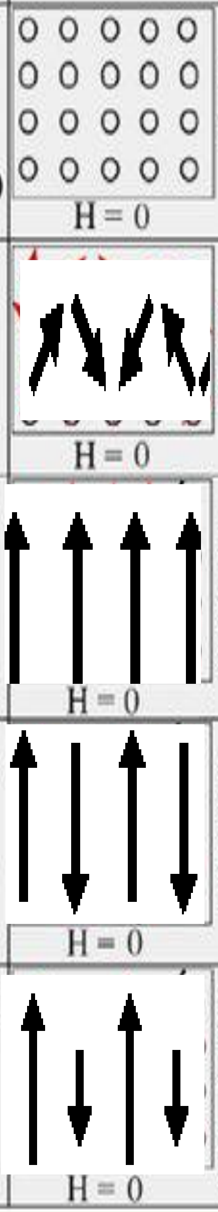
$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

❖ Types of Magnetism

Five basic types of magnetism have been observed and classified based on the magnetic behavior of materials in response to magnetic fields at different temperatures. These types of magnetism are:

1. Ferromagnetism,
2. Ferrimagnetism,
3. Antiferromagnetism,
4. Paramagnetism, and
5. Diamagnetism.

Magnetism	Examples	Magnetic behaviour
Diamagnetism	Bi, Si, Cu, inert gases Susceptibility small and negative (-10^{-6} to -10^{-5})	H = 0 — Atoms have no magnetic moments.
Paramagnetism	Al, O_2, MnBi Susceptibilty small and positive (10^{-5} to 10^{-3})	H = 0 — Atoms have randomly oriented magnetic moments.
Ferromagnetism	Fe, Ni, Co, Gd Susceptibility large (generally > 100)	H = 0 — Atoms are organized in domains which have parallel aligned magnetic moments.
Antiferromagnetism	Cr, MnO, FeO Susceptibilty small and positive (10^{-5} to 10^{-3})	H = 0 — Atoms are organized in domains which have antiparallel aligned moments.
Ferrimagnetism	Fe_3O_4, $MnFe_2O_4$, $NiFe_2O_4$ Susceptibility large (generally > 100)	H = 0 — Atoms are organized in domains which have a mixture of unequal antiparallel aligned moments.



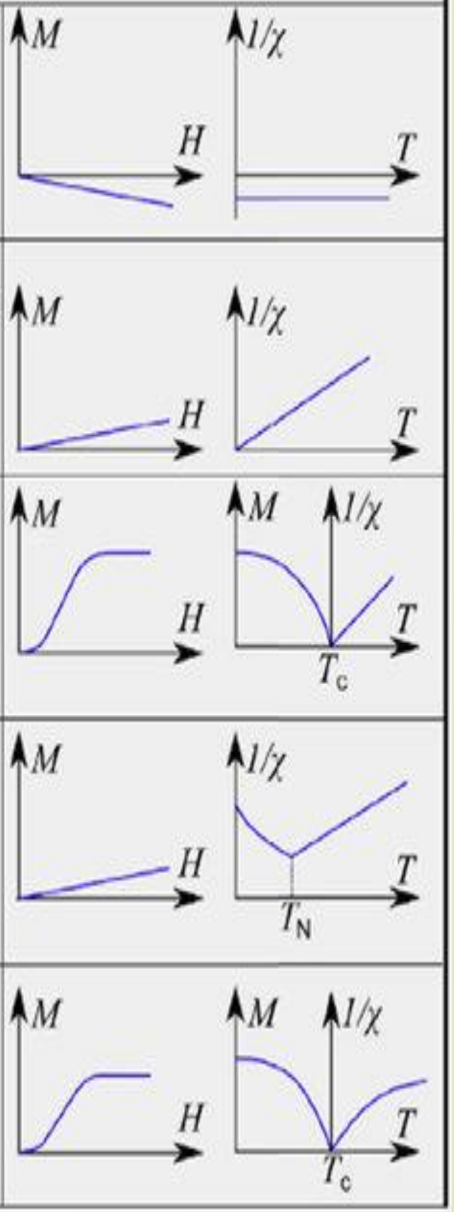
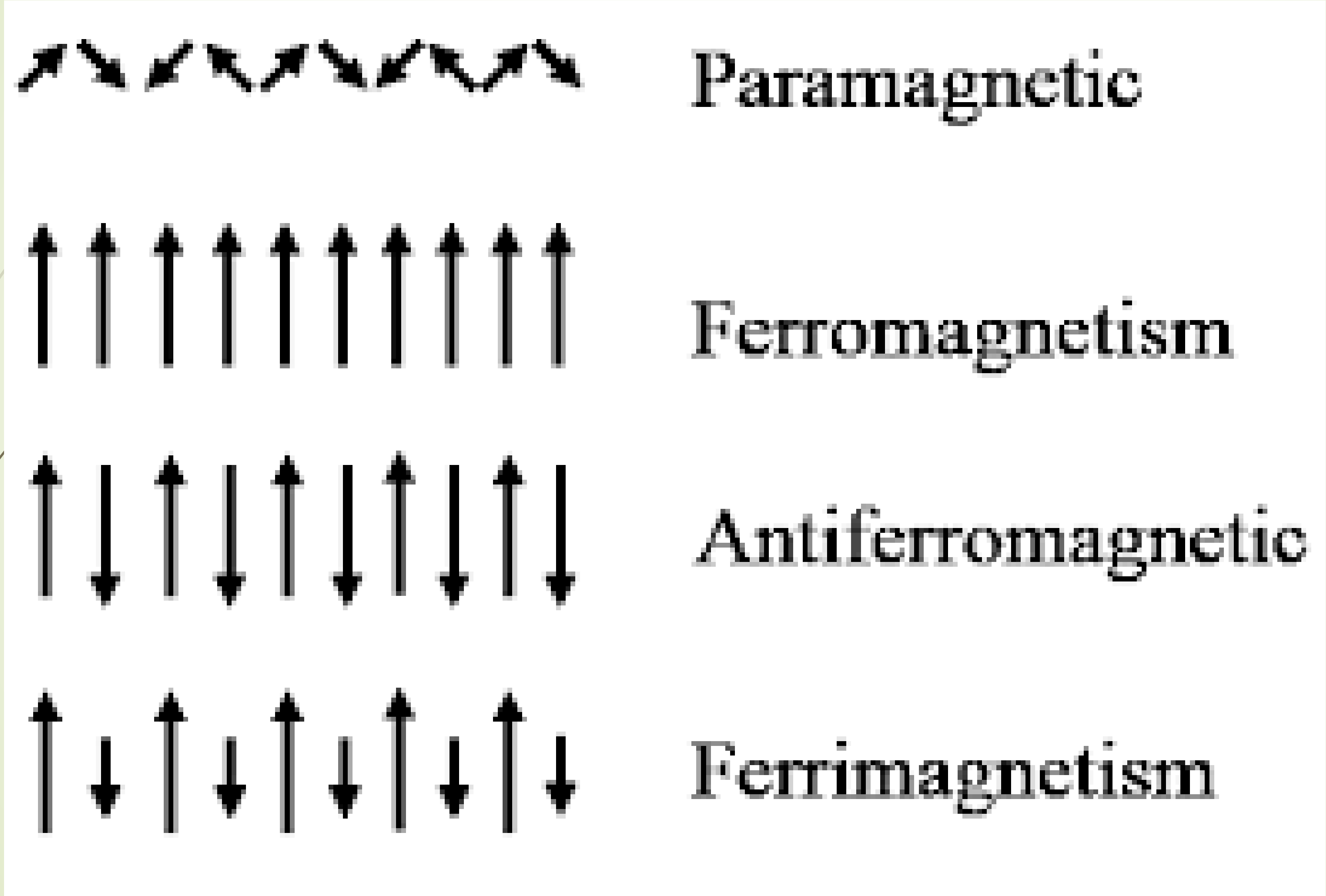


Figure 2.7: Different types of magnetism [56]


Paramagnetic
Ferromagnetism
Antiferromagnetic
Ferrimagnetism

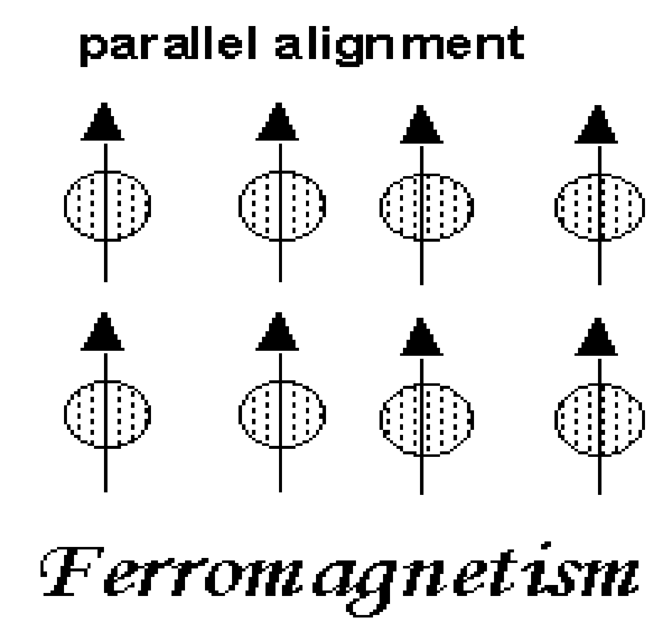
❖ Ferromagnetism

Ferromagnetism is a physical phenomenon whereby some materials, such as iron, attract one other aggressively. The magnetic moments in a ferromagnet have the tendency to become aligned parallel to each other under the influence of a magnetic field. Ferromagnetic materials exhibit parallel alignment of moments resulting in large net magnetization even in the absence of a magnetic field. The elements Fe, Ni, and Co and many of their alloys are typical ferromagnetic materials.

Two distinct characteristics of ferromagnetic materials are them

(1) Spontaneous magnetization and the existence of

(2) Magnetic ordering temperature



❖ Paramagnetism

In a paramagnet, the magnetic moments tend to be randomly orientated due to thermal fluctuations when there is no magnetic field. In an applied magnetic field, these moments start to align parallel to the field such that the magnetization of the material is proportional to the applied field.

❖ Antiferromagnetism

Adjacent magnetic moments from the magnetic ions tend to align anti-parallel to each other without an applied field. In the simplest case, adjacent magnetic moments are equal in magnitude and opposite therefore there is no overall magnetization.

❖ Ferrimagnetism

The aligned magnetic moments are not of the same size; there is more than one type of magnetic ion. An overall magnetization is produced but not all the magnetic moments may give a positive contribution to the overall magnetization.

❖ Diamagnetism

A magnetism is characterized by materials that correspond at right angles toward a non-uniform distribution magnetic field and partially expel the magnetic field in which they have been positioned from their interiors. A diamagnetic substance is one whose atoms have no permanent magnetic dipole moment.

13th Week

Topic: Electricity & Magnetism:

Magnetic Domains and Hysteresis:

Origin of magnetism,

Page: 219-224

Magnetic domains

Magnetic domains are regions within a ferromagnetic or ferrimagnetic material where the magnetic moments of atoms are aligned in the same direction, resulting in a local net magnetic field. These domains arise because of the quantum mechanical exchange interactions between atoms, which favor parallel alignment of adjacent magnetic moments.

Key Features of Magnetic Domains

1. **Aligned Magnetic Moments**:
   1. Within a single domain, all magnetic moments (spins of electrons) are aligned in the same direction, leading to a strong local magnetic field.

2. **Domain Boundaries (Walls)**:
   1. The regions separating different magnetic domains are called **domain walls**.
   2. In these walls, the magnetic moments gradually change orientation to minimize energy between adjacent domains.

3. **Size and Shape**:
   1. Domain size varies depending on the material, temperature, and magnetic field. Domains can range from nanometers to micrometers.

4. **Minimizing Energy**:
   1. The arrangement of domains minimizes the total magnetic energy of the material by balancing:
      1. **Exchange Energy**: Prefers uniform alignment of moments.
      2. **Magnetostatic Energy**: Prefers configurations with low external stray fields.
      3. **Anisotropy Energy**: Depends on the crystal structure and easy axis of magnetization.

Behavior of Magnetic Domains:

1.In the Absence of External Field:

1. Domains are randomly oriented, resulting in no net macroscopic magnetization (material appears non-magnetic).

2.Under an External Magnetic Field:

1. Domains aligned with the field grow at the expense of others through domain wall movement and rotation of moments.
2. This increases the net magnetization of the material.

3.Saturation Magnetization:

1. When all domains are fully aligned with the external magnetic field, the material reaches **saturation magnetization**.

4.Hysteresis:

1. When the magnetic field is removed, some domains remain aligned, leading to residual magnetization, which is the basis of **magnetic memory** in materials.

- ❖ Explain the Hysteresis loop.

A hysteresis loop is a graphical representation of the relationship between two interrelated physical quantities, where the response of one quantity to changes in the other exhibits a lagging or delayed effect.

The loop is generated by measuring the magnetic flux coming out from the ferromagnetic substance while changing the external magnetizing field. If B is measured for various values of H and if the results are plotted in graphic forms, then the graph will show a hysteresis loop.

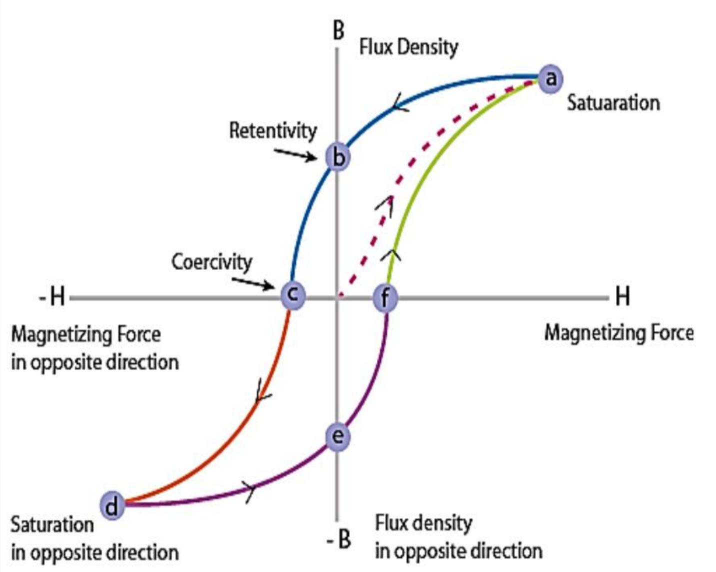
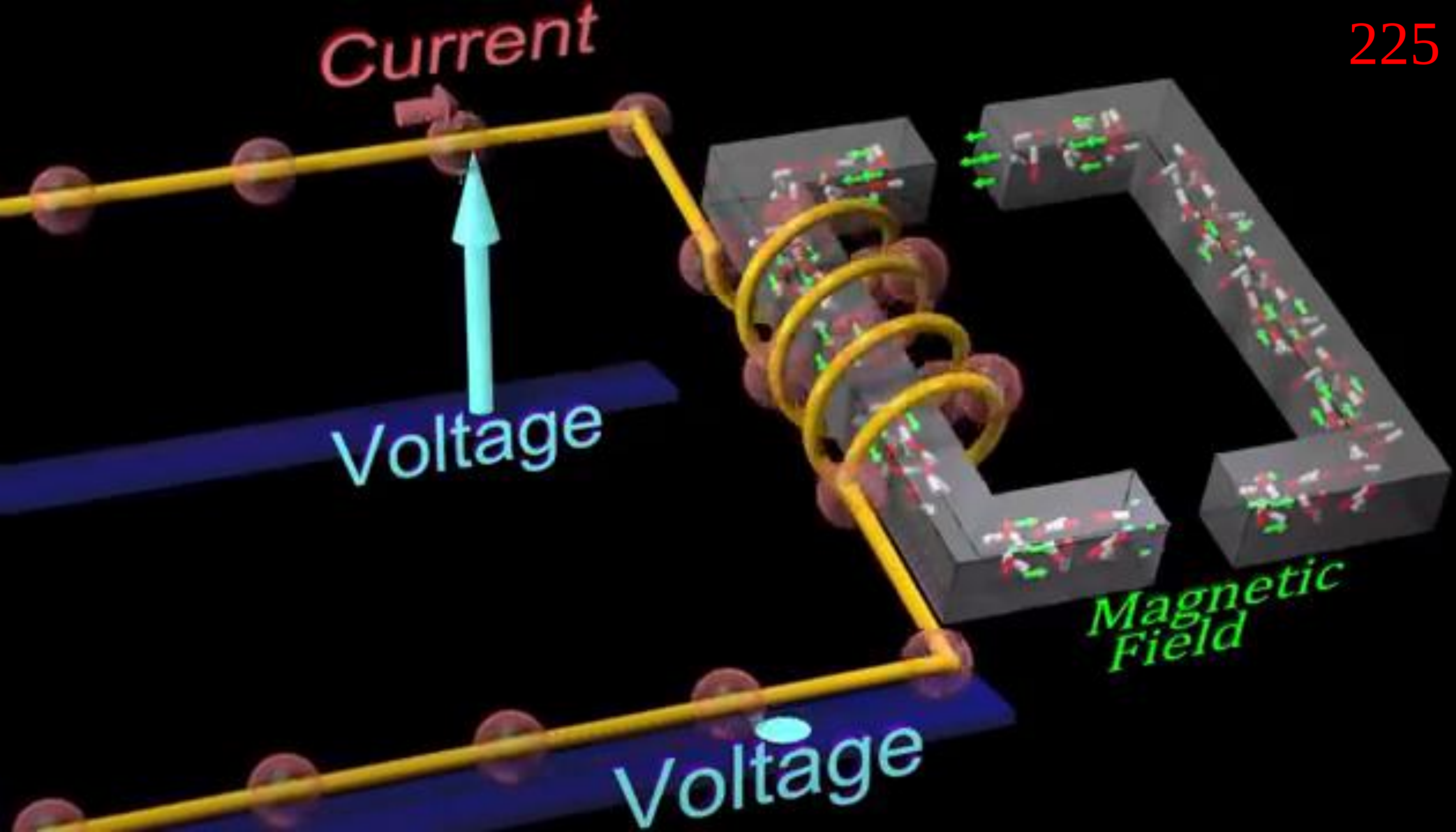


Figure. 2.10 Hysteresis loop of magnetic materials [59].

- ➢ The magnetic flux density (b) is increased when the magnetic field strength (h) is increased from 0 (zero).
- ➢ With an increase in the magnetic field, there is an increase in the value of magnetism, and it finally reaches point a, which is called the saturation point where b is constant.

- With a decrease in the value of the magnetic field, there is a decrease in the value of the magnetism. But if B and H are equal to zero, when a substance or material retains some amount of magnetism, it is called retentivity or residual magnetism.
- When there is a decrease in the magnetic field towards the negative side, magnetism also decreases. At point c, the substance is completely demagnetized.
- The force required to remove the retentivity of the material is known as Coercive force (c).
- In the opposite direction, the cycle is continued where the saturation point is d, the retentivity point is e, and the coercive force is f.

Due to the forward and opposite direction process, the cycle is complete, and this cycle is called the hysteresis loop.


Current
Voltage
Magnetic
Field
Voltage

14TH WEEK

TOPIC: ELECTRICITY & MAGNETISM:

THE BIOT-SAVART LAW, AMPERE'S LAW

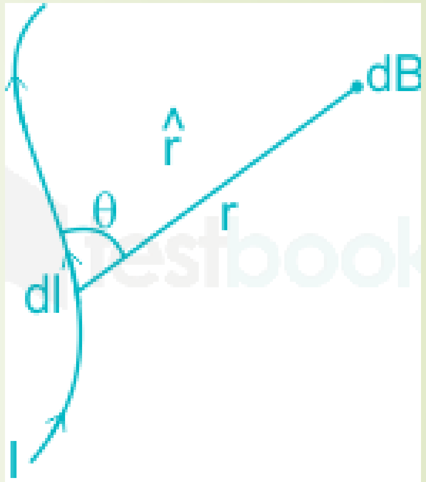
TOPIC RELATED PROBLEMS.

PAGE: 225-231

❖ Describe the Biot savart law:

1. Biot Savart law states that " magnetic field due to a current-carrying conductor at a distance point is inversely proportional to the square of the distance between the conductor and point, and the magnetic field is directly proportional to the length of the conductor, current flowing in the conductor".

2.It can be mathematically expressed as: $dB = \frac{\mu_o}{4\pi}\frac{IdLsin\theta}{r^2}$

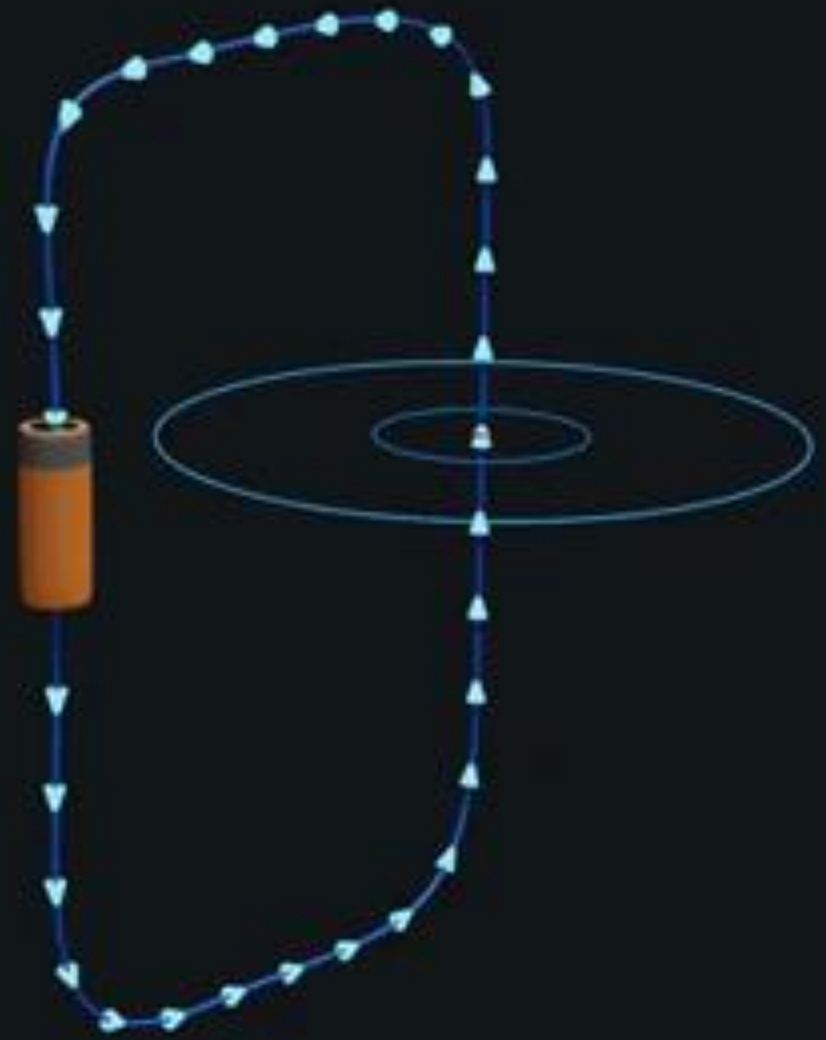


Derivation:

- Consider a point charge P placed at a distance r from an infinitely small length of wire dl.
- The distance vector r makes an angle θ from the direction of the current.
- As the current flow into the wire the current along the wire can be expressed as: dB ∝ I
- It can also be observed that as we move away from wire toward P the magnetic field decreases. i.e dB∝$\frac{1}{r^2}$
- Since θ is the angle between r→ and I.
- so the expression of magnetic field can be given as:

  dB ∝ Idl sinθ
- Combining the equation we get, $dB = K\frac{IdLsin\theta}{r^2}$

  or $dB = \frac{\mu_o}{4\pi}\frac{IdLsin\theta}{r^2}$



The Magnetic Field of a Solenoid

If N is the number of turns in the length ℓ, the total current through the rectangle

this path gives

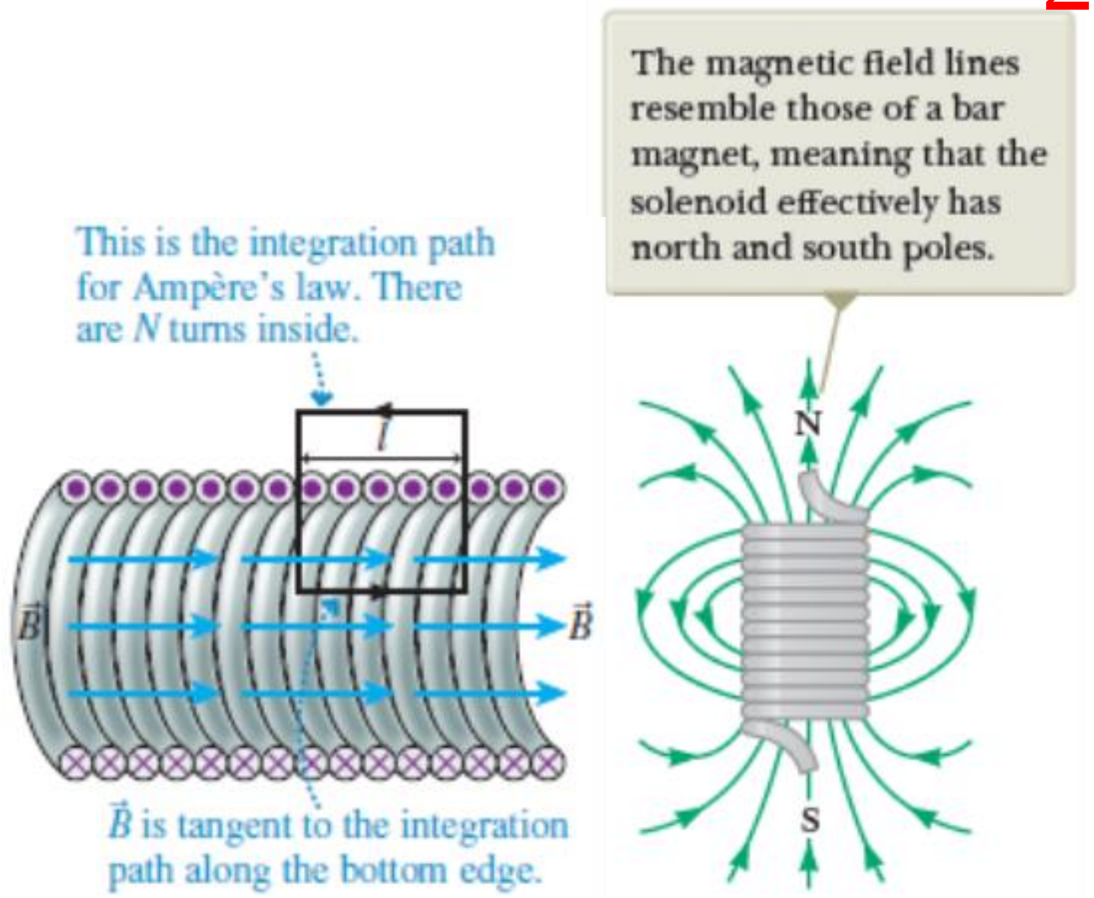
$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}} = B\ell = \mu_0 NI$$

The strength of the uniform magnetic field inside a solenoid is

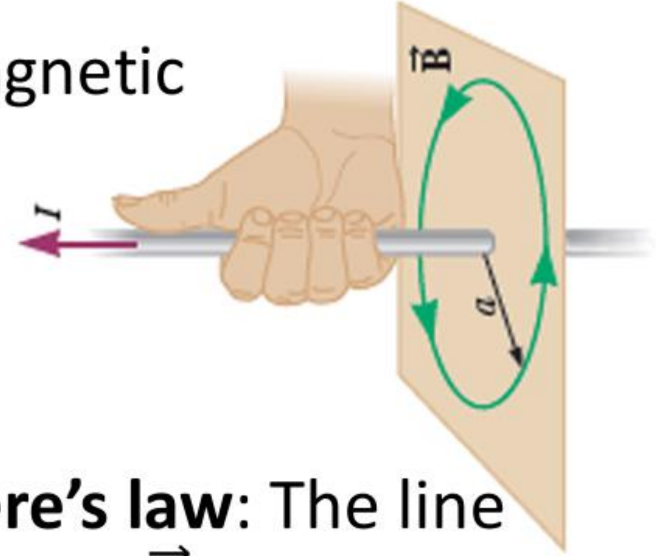
$$B_{solenoid} = \frac{\mu_0 NI}{\ell} = \mu_0 nI$$

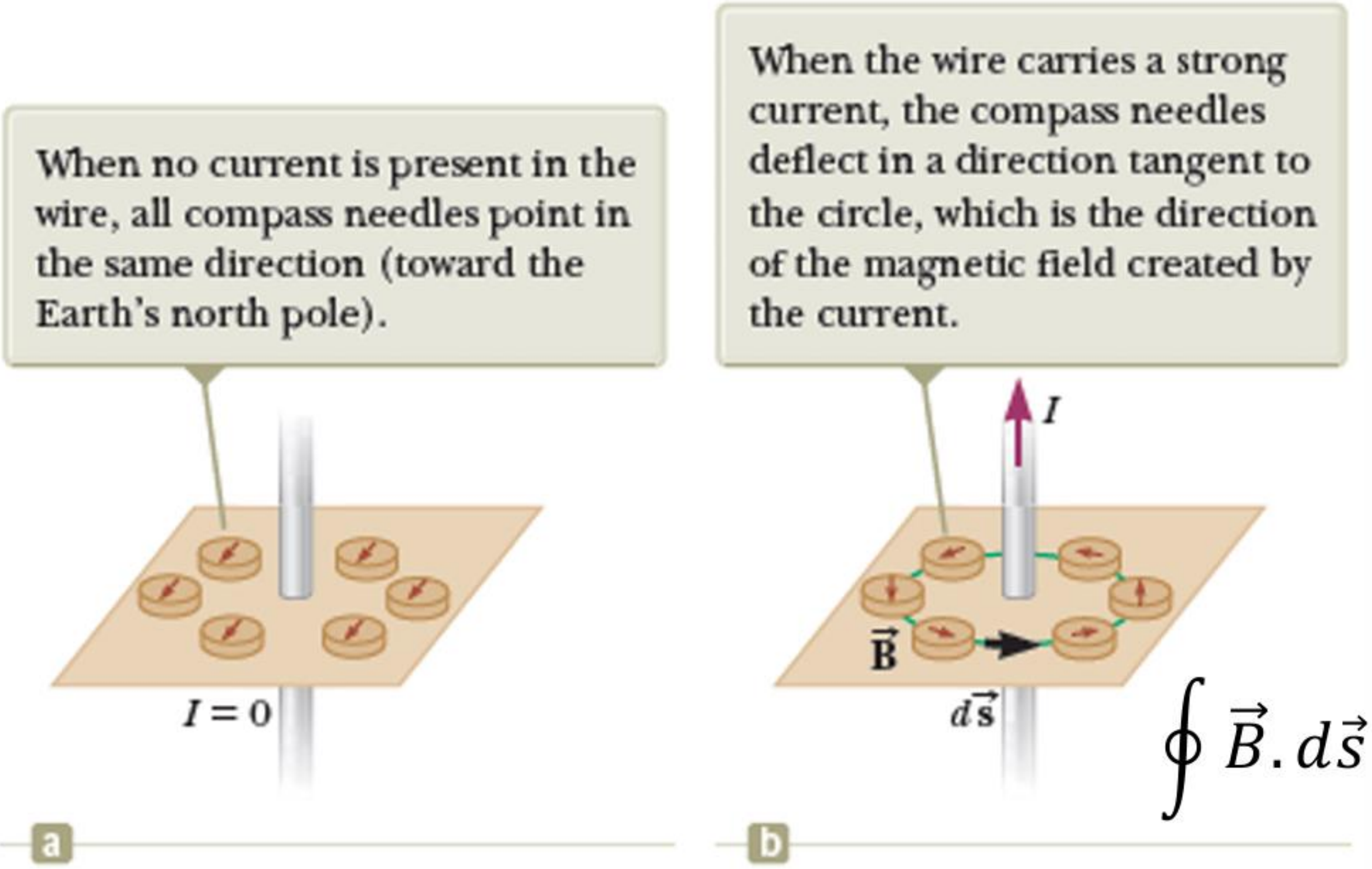
where $n = N/\ell$ is the number of turns per unit length.

Measurements that need a uniform magnetic field are often conducted inside a solenoid, which can be built quite large.



The right-hand rule for determining the direction of the magnetic field surrounding a long, straight wire carrying a current.





Ampère's law: The line integral of $\vec{B}.d\vec{s}$ around any closed path equals $\mu_0 I$, where *I* is the total steady current passing through any surface bounded by the closed path:

$$\oint \vec{B}.d\vec{s} = B\oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

Problem-27

A 1.0-m-long MRI solenoid generates a 1.2 T magnetic field. To produce such a large field, the

that can carry a 100 A current. How many turns of wire does the solenoid need?

Solution:

$$N = \frac{lB}{\mu_0 I} = \frac{(1.0\text{ m})(1.2\text{ T})}{(4\pi \times 10^{-7}\text{ Tm/A})(100\text{ A})} = 9500\text{ turns}$$

Quick Quiz Consider a solenoid that is very long compared with its radius. Of the following choices, what is the most effective way to increase the magnetic field in the interior of the solenoid? **(a)** double its length, keeping the number of turns per unit

length constant **(c)** overwrap the entire solenoid with an additional layer of current-carrying wire

15TH WEEK

TOPIC: ELECTRICITY & MAGNETISM

MAXWELL'S FUNDAMENTAL LAWS, TOPIC RELATED PROBLEMS

PAGE: 232-235

❖ Maxwell's fundamental laws

Maxwell's fundamental laws, also known as Maxwell's equations, are a set of four fundamental equations in classical electromagnetism that describe the behavior of electric and magnetic fields. The four Maxwell's equations are as follows:

1. Gauss's Law for Electricity:

$$\nabla \cdot E = \rho / \varepsilon_0$$

This equation relates the electric field (E) to the charge density (ρ) within a given region. It states that the total electric flux through a closed surface (given by the divergence of the electric field) is proportional to the total charge enclosed by that surface, with ε_0 being the vacuum permittivity (electric constant).

2. Gauss's Law for Magnetism:

$$\nabla \cdot B = 0$$

This equation relates the magnetic field (B) to the magnetic flux within a given region. Unlike electric charges, there are no magnetic monopoles (single magnetic charges) observed in nature. Therefore, the magnetic flux through any closed surface is always zero, and there are no isolated magnetic poles. This is why the divergence of the magnetic field is zero.

3. Faraday's Law of Electromagnetic Induction:

$$\nabla \times E = -\partial B/\partial t$$

This equation expresses how a time-varying magnetic field (B) induces an electric field (E). It states that the curl of the electric field is equal to the negative rate of change of the magnetic field with respect to time. This law is fundamental to understanding electromagnetic induction, which is the basis for generating electric currents in coils and transformers.

4. Ampère's Law with Maxwell's Addition:

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \, \partial E/\partial t$$

This equation describes the relationship between the magnetic field (B) and the electric current density (J) within a given region. It states that the curl of the magnetic field is proportional to the sum of the current density and the rate of change of the electric field with respect to time. The term μ_0 represents the vacuum permeability (magnetic constant).

For More Information & Mathematical Problems See the Book : B.Sc Physics Volume - 1 by C.L Arora

16 &17th Week

Topic: Review the whole Topic of this Course


Any Question ?

The End


thank you
danke
謝謝
ngiyabonga
شكراً جزيلاً
рахмат
teşekkür ederim
Баярлалаа
спасибо
mersi
kia ora
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tapadh leat
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